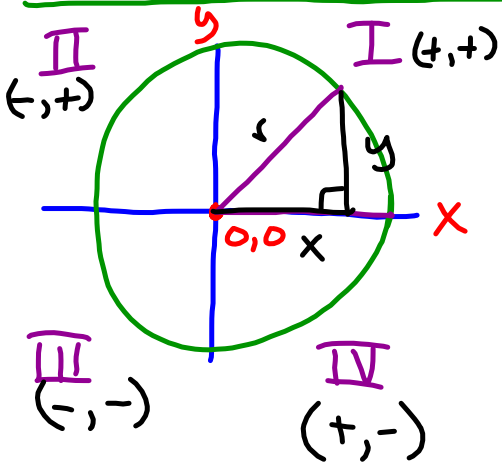
Section 4.3A: The Unit Circle



$r = \sqrt{x^2+y^2} \geq 0$

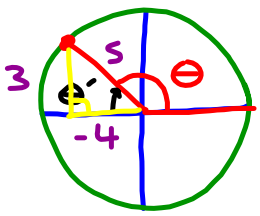
r is take the place or H

$\sin\theta = \frac{y}{r}$ $\quad \csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\quad \sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$

a) $(-4,3)$ is a point on the terminal side of an angle in standard position. Find the value of the 6 trig functions.

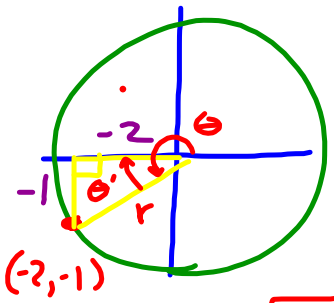


$\sin\theta = \frac{3}{5}$ $\quad \csc\theta = \frac{5}{3}$

$\cos\theta = -\frac{4}{5}$ $\quad \sec\theta = -\frac{5}{4}$

$\tan\theta = -\frac{3}{4}$ $\quad \cot\theta = -\frac{4}{3}$

b) $(-2,-1)$



$r = \sqrt{(-1)^2+(-2)^2}$

$r = \sqrt{5}$

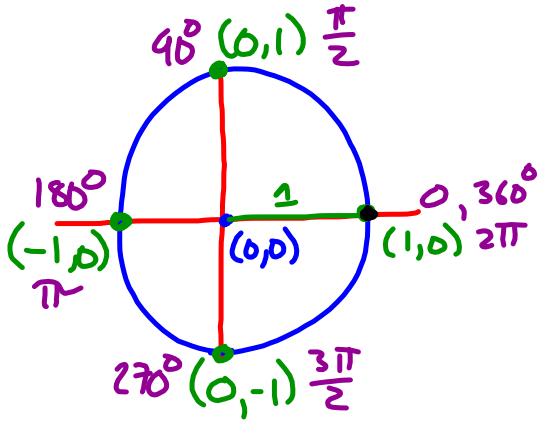
$\sin\theta = -\frac{\sqrt{5}}{5}$ $\quad \csc\theta = -\sqrt{5}$

$\cos\theta = -\frac{2\sqrt{5}}{5}$ $\quad \sec\theta = -\frac{\sqrt{5}}{2}$

$\tan\theta = \frac{1}{2}$ $\quad \cot\theta = 2$

$\sin\theta = \frac{y}{r} = -\frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$

The Unit Circle (r=1)



Quadrantal Angles
(on axes, Flat triangles)

$\pm 0°, 90°, 180°, 270°, 360°, 450°, \ldots$

$\pm 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}, \ldots$

$r = 1$

$\sin\theta = y \quad \csc\theta = \frac{1}{y}$

$\cos\theta = x \quad \sec\theta = \frac{1}{x}$

$\tan\theta = \frac{y}{x} \quad \cot\theta = \frac{x}{y}$

$\sin(0) = 0 \quad \csc(0) = \frac{1}{0} = u$

$\cos(0) = 1 \quad \sec(0) = 1$

$\tan(0) = 0 \quad \cot(0) = u$

$\sin(90°) = 1 \quad \csc(90°) = 1$

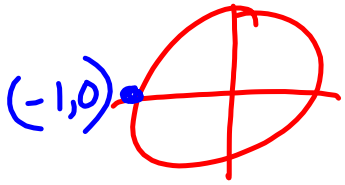
$\cos(90°) = 0 \quad \sec(90°) = u$

$\tan(90°) = u \quad \cot(90°) = 0$

$\sin \pi = 0$ $\csc \pi = \text{U}$

$\cos \pi = -1$ $\sec \pi = -1$

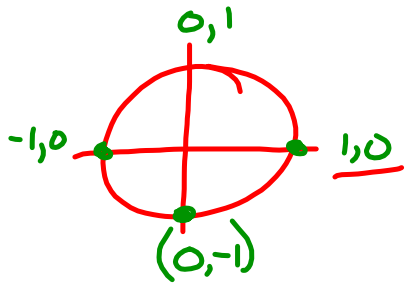
$\tan \pi = 0$ $\cot \pi = \text{U}$



$\sin \frac{3\pi}{2} = -1$ $\csc \frac{3\pi}{2} = -1$

$\cos \frac{3\pi}{2} = 0$ $\sec \frac{3\pi}{2} = \text{U}$

$\tan \frac{3\pi}{2} = \text{U}$ $\cot \frac{3\pi}{2} = 0$

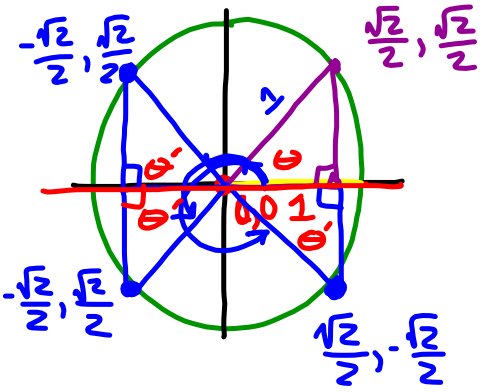


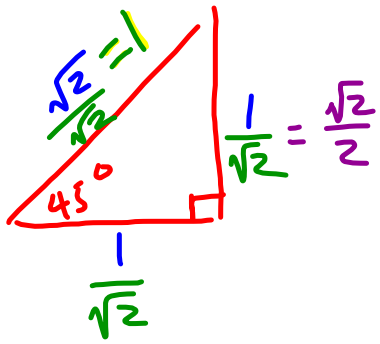
$\sin/\cos/\tan 360^\circ = \sin/\cos/\tan 0^\circ$

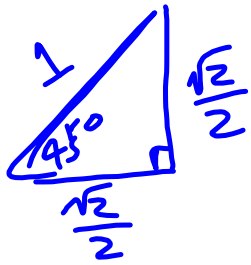
$\sin/\cos/\tan \frac{5\pi}{2} = \sin/\cos/\tan \frac{\pi}{2}$

because they use the same point(s)

We can use this same idea with other angles!







$\sin 45^\circ = \sin 135^\circ = \sin 225^\circ = \sin 315^\circ$
$= \sin 405^\circ =$ except for sign changes

all these have a reference angle $= 45^\circ$

θ' is always the angle to the x-axis
horizontal

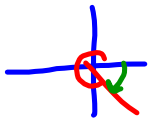
I $\theta = \theta'$

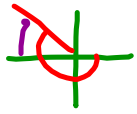
II $180^\circ - \theta = \theta'$

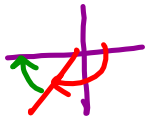
III $\theta - 180^\circ = \theta'$

IV $360^\circ - \theta = \theta'$

Find the reference angles

a) 300°  60°

b) -240°  60°

c) $-\frac{2\pi}{3}$  $60^\circ = \frac{\pi}{3}$

d) $\frac{5\pi}{4}$  $45^\circ = \frac{\pi}{4}$

Signs of Trig Functions

+	+
−	−

$\sin = y$ (csc)

−	+
−	+

$\cos = x$ (sec)

−	+
+	−

$\tan = \frac{y}{x}$ (cot)

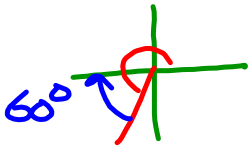
where are the trig functions positive?

S	A
sin/csc	all trig functions are pos.
T	C
tan/cot	cos/sec

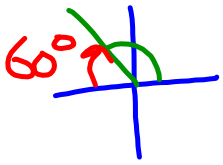
All Student Take Calculus

1-39 odd (4.3)

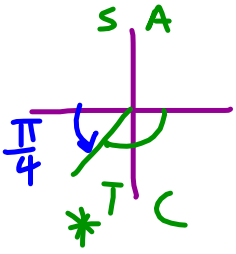
$\sin 240^\circ = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$



$\cos 120^\circ = -\cos 60^\circ = -\frac{1}{2}$



$\tan -\frac{3\pi}{4} = \tan \frac{\pi}{4} = 1$



steps: 1) determine quadrant & sign of trig function
2) find the reference angle
3) put 1&2 together to find the value of the function