37. LHS $= \dfrac{\sin x}{\cos x} - \dfrac{\sin y}{\cos y} = \dfrac{\sin x \cos y - \cos x \sin y}{\cos x \cos y} =$ RHS

38. LHS $= 1 - \dfrac{\sin x \sin y}{\cos x \cos y} = \dfrac{\cos x \cos y - \sin x \sin y}{\cos x \cos y} =$ RHS

39. LHS $= \dfrac{\sin x \cos y + \cos x \sin y - (\sin x \cos y - \cos x \sin y)}{\cos x \cos y - \sin x \sin y + \cos x \cos y + \sin x \sin y}$

$= \dfrac{2 \cos x \sin y}{2 \cos x \cos y} =$ RHS

40. LHS $= (\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)$

$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$

$= \cos^2 x (1 - \sin^2 y) - (1 - \cos^2 x) \sin^2 y$

$= \cos^2 x - \sin^2 y \cos^2 x + \sin^2 y \cos^2 x - \sin^2 y =$ RHS

41. LHS $= \sin((x + y) + z)$

$= \sin(x + y) \cos z + \cos(x + y) \sin z$

$= \cos z[\sin x \cos y + \cos x \sin y]$

$+ \sin z[\cos x \cos y - \sin x \sin y] =$ RHS

42. LHS $= \tan(x - y + y - z)[1 - \tan(x - y) \tan(y - z)]$

$+ \tan(z - x)$

$= \tan(x - z)[1 - \tan(x - y) \tan(y - z)]$

$+ \tan(z - x)$

$= \tan(x - z) + \tan(z - x) - \tan(x - y)$

$\times \tan(y - z) \tan(x - z)$

$= \tan(x - z) - \tan(x - z) - \tan(x - y)$

$\times \tan(y - z) \tan(x - z)$

$= 0 - \tan(x - y) \tan(y - z) \tan(x - z) =$ RHS

43. $\dfrac{\sqrt{1 - x^2} + xy}{\sqrt{1 + y^2}}$ **44.** $\dfrac{xy + \sqrt{1 - x^2}\sqrt{1 - y^2}}{y\sqrt{1 - x^2} - x\sqrt{1 - y^2}}$

45. $\dfrac{x - y}{\sqrt{1 + x^2}\sqrt{1 + y^2}}$ **46.** $xy + \sqrt{1 - x^2} \cdot \sqrt{1 - y^2}$

47. $\frac{1}{4}(\sqrt{6} + \sqrt{2})$ **48.** 0 **49.** $\dfrac{3 - 2\sqrt{14}}{\sqrt{7} + 6\sqrt{2}}$

50. $\frac{1}{15}(10 - 2\sqrt{5})$ **51.** $-\frac{1}{10}(3 + 4\sqrt{3})$ **52.** $-3\sqrt{10}/10$

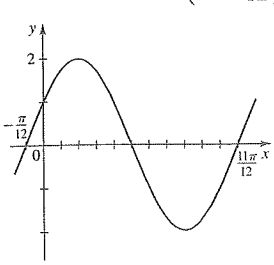
53. $2\sqrt{5}/65$ **54.** $\dfrac{2\sqrt{30} - 1}{\sqrt{15} + 2\sqrt{2}}$ **55.** $2 \sin\left(x + \dfrac{5\pi}{6}\right)$

56. $\sqrt{2} \sin\left(x + \dfrac{\pi}{4}\right)$ **57.** $5\sqrt{2} \sin\left(2x + \dfrac{7\pi}{4}\right)$

58. $6 \sin \pi(x + \frac{1}{3})$

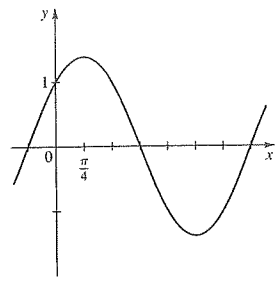
59. (a) $g(x) = 2 \sin 2\left(x + \dfrac{\pi}{12}\right)$

(b)



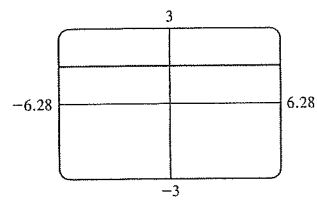
60. (a) $f(x) = \sqrt{2} \sin\left(x + \dfrac{\pi}{4}\right)$

(b)



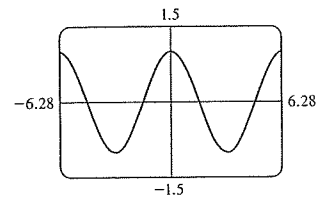
63. $\tan \gamma = \frac{17}{6}$ **64.** (c) $3\pi/4$

65. (a)



$\sin^2\left(x + \dfrac{\pi}{4}\right) + \sin^2\left(x - \dfrac{\pi}{4}\right) = 1$

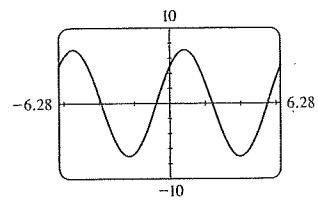
66. (a)



$-\frac{1}{2}[\cos(x + \pi) + \cos(x - \pi)] = \cos x$

67. $\pi/2$

68. (a)



(b) $k = 5\sqrt{2}, \phi = \pi/4$

69. (b) $k = 10\sqrt{3}, \phi = \pi/6$

SECTION 7.3 ■ PAGE 514

1. Double-Angle; $2 \sin x \cos x$

2. Half-Angle; $\pm\sqrt{(1 - \cos x)/2}$

3. $\frac{120}{169}, \frac{119}{169}, \frac{120}{119}$ **4.** $-\frac{24}{25}, -\frac{7}{25}, \frac{24}{7}$ **5.** $-\frac{24}{25}, \frac{7}{25}, -\frac{24}{7}$

6. $-\sqrt{15}/8, \frac{7}{8}, -\sqrt{15}/7$ **7.** $\frac{24}{25}, \frac{7}{25}, \frac{24}{7}$ **8.** $-\sqrt{3}/2, -\frac{1}{2}, \sqrt{3}$

9. $-\frac{3}{5}, \frac{4}{5}, -\frac{3}{4}$ **10.** $\frac{12}{13}, -\frac{5}{13}, -\frac{12}{5}$ **11.** $\frac{1}{2}(\frac{3}{4} - \cos 2x + \frac{1}{4}\cos 4x)$

12. $\frac{1}{2}(\frac{3}{4} + \cos 2x + \frac{1}{4}\cos 4x)$

13. $\frac{1}{16}(1 - \cos 2x - \cos 4x + \cos 2x \cos 4x)$

14. $\frac{1}{16}(1 - \cos 4x + \cos 2x - \cos 2x \cos 4x)$

15. $\frac{1}{32}(\frac{3}{4} - \cos 4x + \frac{1}{4}\cos 8x)$

16. $\frac{1}{16}(3 + 7 \cos 2x + \cos 4x + \cos 2x \cos 4x)$

17. $\frac{1}{2}\sqrt{2 - \sqrt{3}}$ **18.** $2 - \sqrt{3}$ **19.** $\sqrt{2} - 1$

20. $\frac{1}{2}\sqrt{2+\sqrt{3}}$ **21.** $-\frac{1}{2}\sqrt{2+\sqrt{3}}$ **22.** $-\frac{1}{2}\sqrt{2-\sqrt{2}}$
23. $\sqrt{2}-1$ **24.** $\frac{1}{2}\sqrt{2-\sqrt{2}}$ **25.** $\frac{1}{2}\sqrt{2+\sqrt{3}}$
26. $2+\sqrt{3}$ **27.** $-\frac{1}{2}\sqrt{2-\sqrt{2}}$ **28.** $\frac{1}{2}\sqrt{2-\sqrt{3}}$
29. (a) $\sin 36°$ (b) $\sin 6\theta$
30. (a) $\tan 14°$ (b) $\tan 14\theta$
31. (a) $\cos 68°$ (b) $\cos 10\theta$
32. (a) $\cos\theta$ (b) $\sin\theta$
33. (a) $\tan 4°$ (b) $\tan 2\theta$
34. (a) $\sin 15°$ (b) $\sin 4\theta$
37. $\sqrt{10}/10, 3\sqrt{10}/10, \frac{1}{3}$ **38.** $3\sqrt{10}/10, -\sqrt{10}/10, -3$
39. $\sqrt{(3+2\sqrt{2})/6}, \sqrt{(3-2\sqrt{2})/6}, 3+2\sqrt{2}$
40. $\frac{1}{2}\sqrt{2-\sqrt{2}}, \frac{1}{2}\sqrt{2+\sqrt{2}}, \sqrt{2}-1$
41. $\sqrt{6}/6, -\sqrt{30}/6, -\sqrt{5}/5$
42. $\frac{1}{2}\sqrt{(26+5\sqrt{26})/13}$,
$-\frac{1}{2}\sqrt{(26-5\sqrt{26})/13}, -5-\sqrt{26}$

43. $\dfrac{2x}{1+x^2}$ **44.** $\dfrac{2x\sqrt{1-x^2}}{2x^2-1}$ **45.** $\sqrt{\dfrac{1-x}{2}}$ **46.** $1-2x^2$

47. $\frac{336}{625}$ **48.** $-\frac{119}{169}$ **49.** $\frac{8}{7}$ **50.** $\sqrt{5}/5$ **51.** $\frac{7}{25}$ **52.** $\sqrt{26}/26$

53. $-8\sqrt{3}/49$ **54.** $-\frac{24}{7}$ **55.** $\frac{1}{2}(\sin 5x - \sin x)$

56. $\frac{1}{2}(\cos 4x - \cos 6x)$ **57.** $\frac{1}{2}(\sin 5x + \sin 3x)$

58. $\frac{1}{2}(\cos 8x + \cos 2x)$ **59.** $\frac{3}{2}(\cos 11x + \cos 3x)$

60. $\frac{11}{2}\left(\sin\dfrac{3x}{4} + \sin\dfrac{x}{4}\right)$ **61.** $2\sin 4x\cos x$

62. $-2\cos\dfrac{5x}{2}\sin\dfrac{3x}{2}$ **63.** $2\sin 5x\sin x$ **64.** $2\cos\dfrac{11x}{2}\cos\dfrac{7x}{2}$

65. $-2\cos\frac{9}{2}x\sin\frac{5}{2}x$ **66.** $2\sin\dfrac{7x}{2}\cos\dfrac{x}{2}$

67. $(\sqrt{2}+\sqrt{3})/2$ **68.** $\frac{3}{4}(\sqrt{2}+\sqrt{3})$ **69.** $\frac{1}{4}(\sqrt{2}-1)$
70. $\sqrt{6}/2$ **71.** $\sqrt{2}/2$ **72.** $\sqrt{6}/2$
73. LHS $= \cos(2\cdot 5x) =$ RHS
74. LHS $= \sin(2\cdot 4x) =$ RHS
75. LHS $= \sin^2 x + 2\sin x\cos x + \cos^2 x$
$= 1 + 2\sin x\cos x =$ RHS

76. LHS $= \dfrac{2\tan x}{\sec^2 x} = 2\cdot\dfrac{\sin x}{\cos x}\cos^2 x = 2\sin x\cos x =$ RHS

77. LHS $= \dfrac{2\sin 2x\cos 2x}{\sin x} = \dfrac{2(2\sin x\cos x)(\cos 2x)}{\sin x} =$ RHS

78. LHS $= \dfrac{1+2\sin x\cos x}{2\sin x\cos x} = 1 + \dfrac{1}{2\sin x\cos x} =$ RHS

79. LHS $= \dfrac{2(\tan x - \cot x)}{(\tan x+\cot x)(\tan x-\cot x)} = \dfrac{2}{\tan x+\cot x}$
$= \dfrac{2}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}\cdot\dfrac{\sin x\cos x}{\sin x\cos x} = \dfrac{2\sin x\cos x}{\sin^2 x+\cos^2 x}$
$= 2\sin x\cos x =$ RHS

80. LHS $= \dfrac{1}{\tan 2x} = \dfrac{1}{\frac{2\tan x}{1-\tan^2 x}} =$ RHS

81. LHS $= \tan(2x+x) = \dfrac{\tan 2x+\tan x}{1-\tan 2x\tan x}$
$= \dfrac{\frac{2\tan x}{1-\tan^2 x}+\tan x}{1-\frac{2\tan x}{1-\tan^2 x}\tan x}$
$= \dfrac{2\tan x+\tan x(1-\tan^2 x)}{1-\tan^2 x-2\tan x\tan x} =$ RHS

82. LHS $= 4[(\sin^2 x+\cos^2 x)^3 - 3(\sin^4 x\cos^2 x+\sin^2 x\cos^4 x)]$
$= 4[1-3\sin^2 x\cos^2 x(\sin^2 x+\cos^2 x)]$
$= 4 - 12\sin^2 x\cos^2 x = 4 - 3(2\sin x\cos x)^2 =$ RHS

83. LHS $= (\cos^2 x+\sin^2 x)(\cos^2 x-\sin^2 x)$
$= \cos^2 x - \sin^2 x =$ RHS

84. LHS $= \dfrac{1-\cos 2(\frac{x}{2}+\frac{\pi}{4})}{1+\cos 2(\frac{x}{2}+\frac{\pi}{4})} = \dfrac{1-\cos(x+\frac{\pi}{2})}{1+\cos(x+\frac{\pi}{2})}$
$= \dfrac{1-(-\sin x)}{1+(-\sin x)} =$ RHS

85. LHS $= \dfrac{2\sin 3x\cos 2x}{2\cos 3x\cos 2x} = \dfrac{\sin 3x}{\cos 3x} =$ RHS

86. LHS $= \dfrac{2\sin 5x\cos 2x}{-2\sin 5x\sin(-2x)} = \dfrac{\cos 2x}{\sin 2x} =$ RHS

87. LHS $= \dfrac{2\sin 5x\cos 5x}{2\sin 5x\cos 4x} =$ RHS

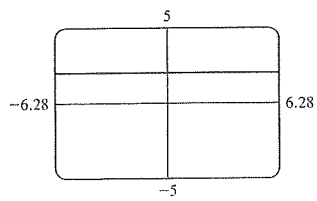
88. $\dfrac{\sin x+\sin 3x+\sin 5x}{\cos x+\cos 3x+\cos 5x} = \dfrac{\sin x+\sin 5x+\sin 3x}{\cos x+\cos 5x+\cos 3x}$
$= \dfrac{2\sin 3x\cos 2x+\sin 3x}{2\cos 3x\cos 2x+\cos 3x} = \dfrac{\sin 3x(2\cos 2x+1)}{\cos 3x(2\cos 2x+1)}$
$= \tan 3x$

89. LHS $= \dfrac{2\sin(\frac{x+y}{2})\cos(\frac{x-y}{2})}{2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})}$
$= \dfrac{\sin(\frac{x+y}{2})}{\cos(\frac{x+y}{2})} =$ RHS

90. $\dfrac{\sin(x+y)-\sin(x-y)}{\cos(x+y)+\cos(x-y)}$
$= \dfrac{2\sin(\frac{x+y+x-y}{2})\cos(\frac{x+y-x+y}{2})}{2\cos(\frac{x+y+x-y}{2})\cos(\frac{x+y-x+y}{2})} = \dfrac{\sin x}{\cos x} = \tan x$

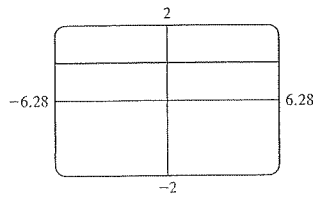
95. LHS $= \dfrac{(\sin x+\sin 5x)+(\sin 2x+\sin 4x)+\sin 3x}{(\cos x+\cos 5x)+(\cos 2x+\cos 4x)+\cos 3x}$
$= \dfrac{2\sin 3x\cos 2x+2\sin 3x\cos x+\sin 3x}{2\cos 3x\cos 2x+2\cos 3x\cos x+\cos 3x}$
$= \dfrac{\sin 3x(2\cos 2x+2\cos x+1)}{\cos 3x(2\cos 2x+2\cos x+1)} =$ RHS

97. (a)



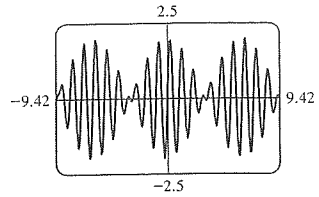
$\dfrac{\sin 3x}{\sin x} - \dfrac{\cos 3x}{\cos x} = 2$

98. (a)

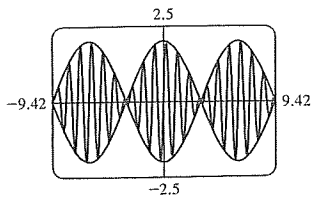


$\cos 2x + 2\sin^2 x = 1$

99. (a)

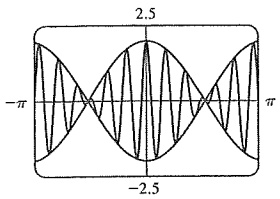


(c)

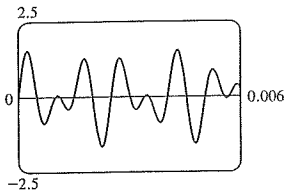


The graph of $y = f(x)$ lies between the two other graphs.

101. (a) $P(t) = 8t^4 - 8t^2 + 1$ (b) $Q(t) = 16t^5 - 20t^3 + 5t$
104. (b) 25 cm^2 (c) 7.07 cm × 3.54 cm
107. (a) and (c)



The graph of f lies between the graphs of $y = 2\cos t$ and $y = -2\cos t$. Thus, the loudness of the sound varies between $y = \pm 2\cos t$.
108. (a) $y = \sin 1540\pi t + \sin 2418\pi t$
(b) $y = 2\sin 1979\pi t \cos 439\pi t$
(c)



SECTION 7.4 ■ PAGE 522

1. infinitely many **2.** no, infinitely many
3. 0.3; $x \approx -9.7, -6.0, -3.4, 0.3, 2.8, 6.6, 9.1$ **4.** (a) 0.30, 2.84
(b) $2\pi, 0.30 + 2k\pi, 2.84 + 2k\pi$ **5.** $\frac{\pi}{3} + 2k\pi, \frac{2\pi}{3} + 2k\pi$
6. $-\frac{\pi}{4} + 2k\pi, \frac{5\pi}{4} + 2k\pi$ **7.** $(2k + 1)\pi$
8. $\frac{\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi$ **9.** $1.32 + 2k\pi, 4.97 + 2k\pi$
10. $-0.30 + 2k\pi, 3.45 + 2k\pi$ **11.** $-0.47 + 2k\pi, 3.61 + 2k\pi$
12. $1.25 + 2k\pi, 5.04 + 2k\pi$ **13.** $-\frac{\pi}{3} + k\pi$ **14.** $\frac{\pi}{4} + k\pi$
15. $1.37 + k\pi$ **16.** $-0.32 + k\pi$ **17.** $\frac{5\pi}{6} + 2k\pi, \frac{7\pi}{6} + 2k\pi$;
$-7\pi/6, -5\pi/6, 5\pi/6, 7\pi/6, 17\pi/6, 19\pi/6$
18. $-\frac{\pi}{3} + 2k\pi, \frac{\pi}{3} + 2k\pi$; $-5\pi/3, -\pi/3, \pi/3, 5\pi/3, 7\pi/3, 11\pi/3$
19. $\frac{\pi}{4} + 2k\pi, \frac{3\pi}{4} + 2k\pi$; $-7\pi/4, -5\pi/4, \pi/4, 3\pi/4, 9\pi/4, 11\pi/4$
20. $-\frac{\pi}{3} + 2k\pi, \frac{4\pi}{3} + 2k\pi$;
$-2\pi/3, -\pi/3, 4\pi/3, 5\pi/3, 10\pi/3, 11\pi/3$
21. $1.29 + 2k\pi, 5.00 + 2k\pi$; $-5.00, -1.29, 1.29, 5.00, 7.57, 11.28$
22. $1.19 + k\pi$; $-5.09, -1.95, 1.19, 4.33, 7.47, 10.61$
23. $-1.47 + k\pi$; $-7.75, -4.61, -1.47, 1.67, 4.81, 7.95$
24. $-1.12 + 2k\pi, 4.26 + 2k\pi$; $-2.02, -1.12, 4.26, 5.16, 10.54, 11.44$ **25.** $(2k + 1)\pi$ **26.** $\frac{3\pi}{2} + 2k\pi$
27. $-\frac{\pi}{4} + 2k\pi, \frac{5\pi}{4} + 2k\pi$ **28.** $-\frac{\pi}{4} + 2k\pi, \frac{\pi}{4} + 2k\pi$
29. $0.20 + 2k\pi, 2.94 + 2k\pi$ **30.** $1.82 + 2k\pi, 4.46 + 2k\pi$
31. $-\frac{\pi}{6} + k\pi, \frac{\pi}{6} + k\pi$ **32.** $\frac{3\pi}{4} + k\pi$ **33.** $\frac{\pi}{4} + k\pi, \frac{3\pi}{4} + k\pi$
34. $\frac{\pi}{3} + k\pi, \frac{2\pi}{3} + k\pi$ **35.** $-1.11 + k\pi, 1.11 + k\pi$
36. $-0.34 + k\pi, 0.34 + k\pi$ **37.** $\frac{\pi}{4} + k\pi, \frac{3\pi}{4} + k\pi$
38. $\frac{\pi}{6} + k\pi, \frac{5\pi}{6} + k\pi$
39. $-1.11 + k\pi, 1.11 + k\pi, \frac{2\pi}{3} + 2k\pi, \frac{4\pi}{3} + 2k\pi$
40. $-0.25 + k\pi, 1.11 + k\pi, 0.25 + k\pi$
41. $\frac{\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi$ **42.** $\frac{7\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi, \frac{\pi}{2} + 2k\pi$
43. $0.34 + 2k\pi, 2.80 + 2k\pi$ **44.** $\pm 1.25 + k\pi, \pm 1.11 + k\pi$
45. $\frac{\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi$ **46.** $\frac{3\pi}{2} + 2k\pi$ **47.** No solution
48. No solution **49.** $\frac{3\pi}{2} + 2k\pi$ **50.** $k\pi, \frac{\pi}{6} + k\pi, \frac{5\pi}{6} + k\pi$
51. $\frac{\pi}{2} + k\pi, \frac{7\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi$
52. $\frac{\pi}{4} + 2k\pi, \frac{7\pi}{4} + 2k\pi$
53. $\frac{\pi}{2} + k\pi$ **54.** $k\pi, \frac{3\pi}{4} + k\pi$
55. $k\pi, 0.73 + 2k\pi, 2.41 + 2k\pi$
56. $\frac{\pi}{2} + k\pi, -0.85 + 2k\pi, 3.99 + 2k\pi$
57. 44.95° **58.** 41.1°
59. (a) 0° (b) 60°, 120° (c) 90°, 270° (d) 180°

SECTION 7.5 ■ PAGE 528

1. $\sin x = 0, k\pi$ **2.** $\sin x + 2\sin x \cos x = 0$,
$\sin x = 0, 1 + 2\cos x = 0$ **3.** $-\frac{\pi}{6} + 2k\pi, \frac{7\pi}{6} + 2k\pi, \frac{\pi}{2} + 2k\pi$
4. No solution **5.** $(2k + 1)\pi, 1.23 + 2k\pi, 5.05 + 2k\pi$
6. $-\frac{\pi}{4} + k\pi, 0.46 + k\pi$ **7.** $k\pi, 0.72 + 2k\pi, 5.56 + 2k\pi$
8. $k\pi, 1.23 + 2k\pi, 5.05 + 2k\pi$ **9.** $\frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$
10. $\frac{\pi}{4} + k\pi, \frac{3\pi}{4} + k\pi$ **11.** $\frac{\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi, (2k + 1)\pi$
12. $\frac{\pi}{3} + k\pi, \frac{2\pi}{3} + k\pi$ **13.** $(2k + 1)\pi, \frac{\pi}{2} + 2k\pi$
14. $2k\pi, \frac{3\pi}{2} + 2k\pi$ **15.** $2k\pi$ **16.** $-1.24 + k\pi, \frac{\pi}{4} + k\pi$