88. $\text{LHS} = \dfrac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{\frac{\cos x}{\sin x} + \frac{\cos y}{\sin y}} = \left(\dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y}\right)$

$\times \left(\dfrac{\sin x \sin y}{\cos x \sin y + \sin x \cos y}\right)$

$= \dfrac{\sin x \sin y}{\cos x \cos y} = \tan x \tan y = \text{RHS}$

89. $\text{LHS} = \left(\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}\right)^4 = \left(\dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right)^4$

$= \left(\dfrac{1}{\sin x \cos x}\right)^4 = \text{RHS}$

90. $\text{LHS} = \left(\sin\alpha - \dfrac{\sin\alpha}{\cos\alpha}\right)\left(\cos\alpha - \dfrac{\cos\alpha}{\sin\alpha}\right)$

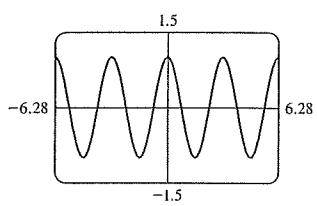
$= \sin\alpha\left(1 - \dfrac{1}{\cos\alpha}\right) \cdot \cos\alpha\left(1 - \dfrac{1}{\sin\alpha}\right)$

$= \cos\alpha\left(1 - \dfrac{1}{\cos\alpha}\right)\sin\alpha\left(1 - \dfrac{1}{\sin\alpha}\right)$

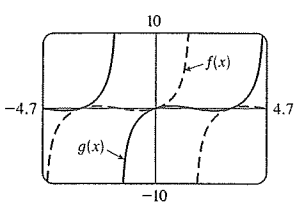
$= (\cos\alpha - 1)(\sin\alpha - 1) = \text{RHS}$

91. $\tan\theta$ **92.** $\sec\theta$ **93.** $\tan\theta$ **94.** $\frac{1}{8}\cot^2\theta\cos\theta$
95. $3\cos\theta$ **96.** $\sin\theta$

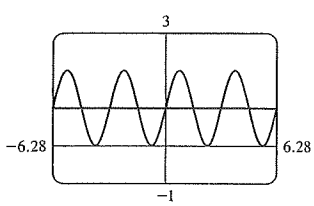
97. Yes



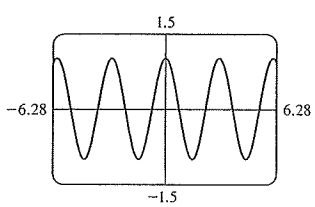
98. No



99. No



100. Yes



SECTION 7.2 ■ PAGE 505

1. addition; $\sin x \cos y + \cos x \sin y$
2. subtraction; $\cos x \cos y + \sin x \sin y$

3. $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ **4.** $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ **5.** $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

6. $-\dfrac{\sqrt{2}+\sqrt{6}}{4}$ **7.** $2-\sqrt{3}$ **8.** $\sqrt{3}-2$

9. $-\dfrac{\sqrt{6}+\sqrt{2}}{4}$ **10.** $\dfrac{\sqrt{2}-\sqrt{6}}{4}$ **11.** $\sqrt{3}-2$

12. $-\dfrac{\sqrt{6}+\sqrt{2}}{4}$ **13.** $-\dfrac{\sqrt{6}+\sqrt{2}}{4}$ **14.** $-(2+\sqrt{3})$

15. $\sqrt{2}/2$ **16.** 0 **17.** $\frac{1}{2}$ **18.** $\sqrt{3}/3$ **19.** $\sqrt{3}$ **20.** $-\frac{1}{2}$

21. $\text{LHS} = \dfrac{\sin(\frac{\pi}{2}-u)}{\cos(\frac{\pi}{2}-u)} = \dfrac{\sin\frac{\pi}{2}\cos u - \cos\frac{\pi}{2}\sin u}{\cos\frac{\pi}{2}\cos u + \sin\frac{\pi}{2}\sin u}$

$= \dfrac{\cos u}{\sin u} = \text{RHS}$

22. $\text{LHS} = \dfrac{\cos(\frac{\pi}{2}-u)}{\sin(\frac{\pi}{2}-u)} = \dfrac{\cos\frac{\pi}{2}\cos u + \sin\frac{\pi}{2}\sin u}{\sin\frac{\pi}{2}\cos u - \cos\frac{\pi}{2}\sin u}$

$= \dfrac{\sin u}{\cos u} = \text{RHS}$

23. $\text{LHS} = \dfrac{1}{\cos(\frac{\pi}{2}-u)} = \dfrac{1}{\cos\frac{\pi}{2}\cos u + \sin\frac{\pi}{2}\sin u}$

$= \dfrac{1}{\sin u} = \text{RHS}$

24. $\text{LHS} = \dfrac{1}{\sin(\frac{\pi}{2}-u)} = \dfrac{1}{\sin\frac{\pi}{2}\cos u - \cos\frac{\pi}{2}\sin u}$

$= \dfrac{1}{\cos u} = \text{RHS}$

25. $\text{LHS} = \sin x \cos\frac{\pi}{2} - \cos x \sin\frac{\pi}{2} = \text{RHS}$
26. $\text{LHS} = \cos x \cos\frac{\pi}{2} + \sin x \sin\frac{\pi}{2} = \text{RHS}$
27. $\text{LHS} = \sin x \cos\pi - \cos x \sin\pi = \text{RHS}$
28. $\text{LHS} = \cos x \cos\pi + \sin x \sin\pi = \text{RHS}$

29. $\text{LHS} = \dfrac{\tan x - \tan\pi}{1 + \tan x \tan\pi} = \text{RHS}$

30. $\text{LHS} = \sin\left(\dfrac{\pi}{2} - x\right) = \sin\dfrac{\pi}{2}\cos x - \cos\dfrac{\pi}{2}\sin x = \cos x$

$\text{RHS} = \sin\left(\dfrac{\pi}{2} + x\right) = \sin\dfrac{\pi}{2}\cos x + \cos\dfrac{\pi}{2}\sin x = \cos x$

31. $\text{LHS} = \cos x \cos\frac{\pi}{6} - \sin x \sin\frac{\pi}{6} + \sin x \cos\frac{\pi}{3} - \cos x \sin\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x + \frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x = \text{RHS}$

32. $\text{LHS} = \dfrac{\tan x - \tan\frac{\pi}{4}}{1 + \tan x \tan\frac{\pi}{4}} = \text{RHS}$

33. $\text{LHS} = \sin x \cos y + \cos x \sin y$

$-(\sin x \cos y - \cos x \sin y) = \text{RHS}$

34. $\text{LHS} = \cos x \cos y - \sin x \sin y + \cos x \cos y$

$+ \sin x \sin y = \text{RHS}$

35. $\text{LHS} = \dfrac{1}{\tan(x-y)} = \dfrac{1 + \tan x \tan y}{\tan x - \tan y}$

$= \dfrac{1 + \frac{1}{\cot x}\frac{1}{\cot y}}{\frac{1}{\cot x} - \frac{1}{\cot y}} \cdot \dfrac{\cot x \cot y}{\cot x \cot y} = \text{RHS}$

36. $\text{LHS} = \dfrac{1}{\tan(x+y)} = \dfrac{1 - \tan x \tan y}{\tan x + \tan y}$

$= \dfrac{1 - \frac{1}{\cot x}\frac{1}{\cot y}}{\frac{1}{\cot x} + \frac{1}{\cot y}} \cdot \dfrac{\cot x \cot y}{\cot x \cot y}$

$= \dfrac{\cot x \cot y - 1}{\cot x + \cot y} = \text{RHS}$

37. LHS $= \dfrac{\sin x}{\cos x} - \dfrac{\sin y}{\cos y} = \dfrac{\sin x \cos y - \cos x \sin y}{\cos x \cos y} =$ RHS

38. LHS $= 1 - \dfrac{\sin x \sin y}{\cos x \cos y} = \dfrac{\cos x \cos y - \sin x \sin y}{\cos x \cos y} =$ RHS

39. LHS $= \dfrac{\sin x \cos y + \cos x \sin y - (\sin x \cos y - \cos x \sin y)}{\cos x \cos y - \sin x \sin y + \cos x \cos y + \sin x \sin y}$

$= \dfrac{2 \cos x \sin y}{2 \cos x \cos y} =$ RHS

40. LHS $= (\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)$

$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$

$= \cos^2 x\,(1 - \sin^2 y) - (1 - \cos^2 x) \sin^2 y$

$= \cos^2 x - \sin^2 y \cos^2 x + \sin^2 y \cos^2 x - \sin^2 y =$ RHS

41. LHS $= \sin((x + y) + z)$

$= \sin(x + y) \cos z + \cos(x + y) \sin z$

$= \cos z[\sin x \cos y + \cos x \sin y]$

$+ \sin z[\cos x \cos y - \sin x \sin y] =$ RHS

42. LHS $= \tan(x - y + y - z)[1 - \tan(x - y) \tan(y - z)]$

$+ \tan(z - x)$

$= \tan(x - z)[1 - \tan(x - y) \tan(y - z)]$

$+ \tan(z - x)$

$= \tan(x - z) + \tan(z - x) - \tan(x - y)$

$\times \tan(y - z) \tan(x - z)$

$= \tan(x - z) - \tan(x - z) - \tan(x - y)$

$\times \tan(y - z) \tan(x - z)$

$= 0 - \tan(x - y) \tan(y - z) \tan(x - z) =$ RHS

43. $\dfrac{\sqrt{1 - x^2} + xy}{\sqrt{1 + y^2}}$ **44.** $\dfrac{xy + \sqrt{1 - x^2}\sqrt{1 - y^2}}{y\sqrt{1 - x^2} - x\sqrt{1 - y^2}}$

45. $\dfrac{x - y}{\sqrt{1 + x^2}\sqrt{1 + y^2}}$ **46.** $xy + \sqrt{1 - x^2} \cdot \sqrt{1 - y^2}$

47. $\frac{1}{4}(\sqrt{6} + \sqrt{2})$ **48.** 0 **49.** $\dfrac{3 - 2\sqrt{14}}{\sqrt{7} + 6\sqrt{2}}$

50. $\frac{1}{15}(10 - 2\sqrt{5})$ **51.** $-\frac{1}{10}(3 + 4\sqrt{3})$ **52.** $-3\sqrt{10}/10$

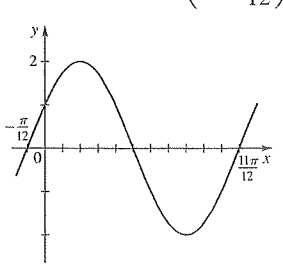
53. $2\sqrt{5}/65$ **54.** $\dfrac{2\sqrt{30} - 1}{\sqrt{15} + 2\sqrt{2}}$ **55.** $2 \sin\left(x + \dfrac{5\pi}{6}\right)$

56. $\sqrt{2} \sin\left(x + \dfrac{\pi}{4}\right)$ **57.** $5\sqrt{2} \sin\left(2x + \dfrac{7\pi}{4}\right)$

58. $6 \sin \pi\left(x + \frac{1}{3}\right)$

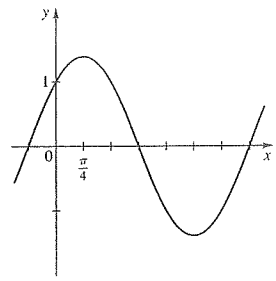
59. (a) $g(x) = 2 \sin 2\left(x + \dfrac{\pi}{12}\right)$

(b)



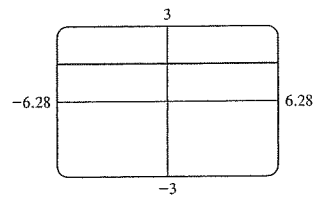
60. (a) $f(x) = \sqrt{2} \sin\left(x + \dfrac{\pi}{4}\right)$

(b)



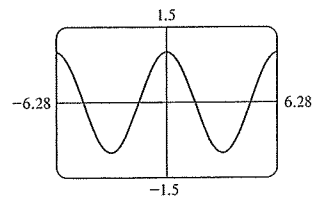
63. $\tan \gamma = \frac{17}{6}$ **64.** (c) $3\pi/4$

65. (a)



$\sin^2\left(x + \dfrac{\pi}{4}\right) + \sin^2\left(x - \dfrac{\pi}{4}\right) = 1$

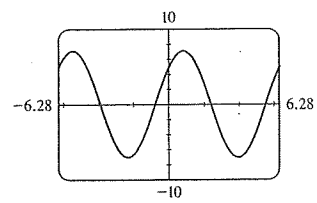
66. (a)



$-\frac{1}{2}[\cos(x + \pi) + \cos(x - \pi)] = \cos x$

67. $\pi/2$

68. (a)



(b) $k = 5\sqrt{2}, \phi = \pi/4$

69. (b) $k = 10\sqrt{3}, \phi = \pi/6$

SECTION 7.3 ■ PAGE 514

1. Double-Angle; $2 \sin x \cos x$

2. Half-Angle; $\pm\sqrt{(1 - \cos x)/2}$

3. $\frac{120}{169}, \frac{119}{169}, \frac{120}{119}$ **4.** $-\frac{24}{25}, -\frac{7}{25}, \frac{24}{7}$ **5.** $-\frac{24}{25}, \frac{7}{25}, -\frac{24}{7}$

6. $-\sqrt{15}/8, \frac{7}{8}, -\sqrt{15}/7$ **7.** $\frac{24}{25}, \frac{7}{25}, \frac{24}{7}$ **8.** $-\sqrt{3}/2, -\frac{1}{2}, \sqrt{3}$

9. $-\frac{3}{5}, \frac{4}{5}, -\frac{3}{4}$ **10.** $\frac{12}{13}, -\frac{5}{13}, -\frac{12}{5}$ **11.** $\frac{1}{2}(\frac{3}{4} - \cos 2x + \frac{1}{4}\cos 4x)$

12. $\frac{1}{2}(\frac{3}{4} + \cos 2x + \frac{1}{4}\cos 4x)$

13. $\frac{1}{16}(1 - \cos 2x - \cos 4x + \cos 2x \cos 4x)$

14. $\frac{1}{16}(1 - \cos 4x + \cos 2x - \cos 2x \cos 4x)$

15. $\frac{1}{32}(\frac{3}{4} - \cos 4x + \frac{1}{4}\cos 8x)$

16. $\frac{1}{16}(3 + 7 \cos 2x + \cos 4x + \cos 2x \cos 4x)$

17. $\frac{1}{2}\sqrt{2 - \sqrt{3}}$ **18.** $2 - \sqrt{3}$ **19.** $\sqrt{2} - 1$