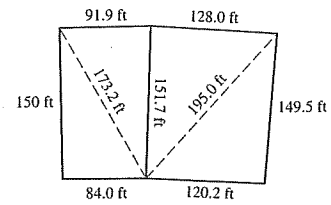
8. $-\frac{13}{12}$ **9.** $\tan\theta = -\sqrt{\sec^2\theta - 1}$ **10.** 19.6 ft
11. (a) $\theta = \tan^{-1}(x/4)$ (b) $\theta = \cos^{-1}(3/x)$ **12.** $\frac{40}{41}$
13. 9.1 **14.** 250.5 **15.** 8.4 **16.** 19.5 **17.** 78.6° **18.** 40.2°
19. (a) 15.3 m^2 (b) 24.3 m **20.** (a) 129.9° (b) 44.9
21. 554 ft

FOCUS ON MODELING ■ PAGE 490

1. 1.41 mi **2.** 1.31 mi
3. 14.3 m **4.** 119.2 m
5. (c) 2349.8 ft **6.** 4194 ft
7.



CHAPTER 7

SECTION 7.1 ■ PAGE 498

1. all; 1 **2.** $\cos(-x) = \cos x$ **3.** $\sin t$ **4.** $\cot t$ **5.** $\tan\theta$
6. $\sec\theta$ **7.** -1 **8.** $\tan x$ **9.** $\csc u$ **10.** 1 **11.** $\tan\theta$
12. $\sec\theta$ **13.** 1 **14.** $\cos x$ **15.** $\cos y$ **16.** $\sin x$ **17.** $\sin^2 x$
18. $\sin x$ **19.** $\sec x$ **20.** 1 **21.** $2\sec u$ **22.** 1 **23.** $\cos^2 x$
24. $\sin A + \cos A$ **25.** $\cos\theta$ **26.** $1 - \sin x$

27. (a) $\text{LHS} = \dfrac{1 - \sin^2 x}{\sin x} = \text{RHS}$

28. (a) $\text{LHS} = \dfrac{1 - \cos^2 y}{\cos y} = \text{RHS}$

29. $\text{LHS} = \sin\theta\dfrac{\cos\theta}{\sin\theta} = \text{RHS}$

30. $\text{LHS} = \dfrac{\sin x}{\cos x}\cdot\cos x = \text{RHS}$

31. $\text{LHS} = \cos u\dfrac{1}{\cos u}\cot u = \text{RHS}$

32. $\text{LHS} = \dfrac{\cos x}{\sin x}\cdot\dfrac{1}{\cos x}\cdot\sin x = \text{RHS}$

33. $\text{LHS} = \sin B + \cos B\dfrac{\cos B}{\sin B}$
$= \dfrac{\sin^2 B + \cos^2 B}{\sin B} = \dfrac{1}{\sin B} = \text{RHS}$

34. $\text{LHS} = \cos x - (-\sin x) = \text{RHS}$

35. $\text{LHS} = -\dfrac{\cos\alpha}{\sin\alpha}\cos\alpha - \sin\alpha = \dfrac{-\cos^2\alpha - \sin^2\alpha}{\sin\alpha}$
$= \dfrac{-1}{\sin\alpha} = \text{RHS}$

36. $\text{LHS} = \csc^2 x - \sin x\csc x = \csc^2 x - 1 = \text{RHS}$

37. $\text{LHS} = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$
$= \dfrac{1}{\cos\theta\sin\theta} = \text{RHS}$

38. $\text{LHS} = \sin^2 x + 2\sin x\cos x + \cos^2 x = \text{RHS}$

39. $\text{LHS} = 1 - \cos^2\beta = \sin^2\beta = \text{RHS}$

40. $\text{LHS} = \cos^2 x + \sin^2 x = \text{RHS}$

41. $\text{LHS} = \dfrac{(\sin x + \cos x)^2}{(\sin x + \cos x)(\sin x - \cos x)} = \dfrac{\sin x + \cos x}{\sin x - \cos x}$
$= \dfrac{(\sin x + \cos x)(\sin x - \cos x)}{(\sin x - \cos x)(\sin x - \cos x)} = \text{RHS}$

42. $\text{LHS} = [(\sin x + \cos x)^2]^2$
$= (\sin^2 x + 2\sin x\cos x + \cos^2 x)^2 = \text{RHS}$

43. $\text{LHS} = \dfrac{\frac{1}{\cos t} - \cos t}{\frac{1}{\cos t}}\cdot\dfrac{\cos t}{\cos t} = \dfrac{1 - \cos^2 t}{1} = \text{RHS}$

44. $\text{LHS} = \dfrac{1 - \sin x}{1 + \sin x}\cdot\dfrac{1 - \sin x}{1 - \sin x} = \dfrac{1 - 2\sin x + \sin^2 x}{1 - \sin^2 x}$
$= \dfrac{1}{\cos^2 x} - \dfrac{2\sin x}{\cos^2 x} + \dfrac{\sin^2 x}{\cos^2 x}$
$= \sec^2 x - 2\sec x\tan x + \tan^2 x$
$= (\sec x - \tan x)^2 = \text{RHS}$

45. $\text{LHS} = \dfrac{1}{\cos^2 y} = \sec^2 y = \text{RHS}$

46. $\text{LHS} = \dfrac{1}{\sin x} - \sin x = \dfrac{1 - \sin^2 x}{\sin x} = \dfrac{\cos^2 x}{\sin x} = \text{RHS}$

47. $\text{LHS} = \cot x\cos x + \cot x - \csc x\cos x - \csc x$
$= \dfrac{\cos^2 x}{\sin x} + \dfrac{\cos x}{\sin x} - \dfrac{\cos x}{\sin x} - \dfrac{1}{\sin x} = \dfrac{\cos^2 x - 1}{\sin x}$
$= \dfrac{-\sin^2 x}{\sin x} = \text{RHS}$

48. $\text{LHS} = (\sin^2\theta)^2 - (\cos^2\theta)^2$
$= (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta) = \text{RHS}$

49. $\text{LHS} = \sin^2 x\left(1 + \dfrac{\cos^2 x}{\sin^2 x}\right) = \sin^2 x + \cos^2 x = \text{RHS}$

50. $\text{LHS} = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1 = \text{RHS}$

51. $\text{LHS} = 2(1 - \sin^2 x) - 1 = 2 - 2\sin^2 x - 1 = \text{RHS}$

52. $\text{LHS} = \left(\dfrac{\sin y}{\cos y} + \dfrac{\cos y}{\sin y}\right)\sin y\cos y$
$= \sin^2 y + \cos^2 y = \text{RHS}$

53. $\text{LHS} = \dfrac{1 - \cos\alpha}{\sin\alpha}\cdot\dfrac{1 + \cos\alpha}{1 + \cos\alpha}$
$= \dfrac{1 - \cos^2\alpha}{\sin\alpha(1 + \cos\alpha)} = \dfrac{\sin^2\alpha}{\sin\alpha(1 + \cos\alpha)} = \text{RHS}$

54. $\text{LHS} = 1 + \tan^2\alpha = \sec^2\alpha = \text{RHS}$

55. $\text{LHS} = \dfrac{\sin^2\theta}{\cos^2\theta} - \dfrac{\sin^2\theta\cos^2\theta}{\cos^2\theta}$
$= \dfrac{\sin^2\theta(1 - \cos^2\theta)}{\cos^2\theta} = \dfrac{\sin^2\theta\sin^2\theta}{\cos^2\theta} = \text{RHS}$

56. $\text{LHS} = \dfrac{\cos^2\theta}{\sin^2\theta}\cos^2\theta = \dfrac{\cos^2\theta(1 - \sin^2\theta)}{\sin^2\theta}$
$= \dfrac{\cos^2\theta}{\sin^2\theta} - \dfrac{\cos^2\theta\sin^2\theta}{\sin^2\theta} = \text{RHS}$

57. $\text{LHS} = \dfrac{\sin x - 1}{\sin x + 1}\cdot\dfrac{\sin x + 1}{\sin x + 1} = \dfrac{\sin^2 x - 1}{(\sin x + 1)^2} = \text{RHS}$

58. $\text{LHS} = \dfrac{\sin w}{\sin w + \cos w}\cdot\dfrac{\frac{1}{\cos w}}{\frac{1}{\cos w}} = \dfrac{\frac{\sin w}{\cos w}}{\frac{\sin w}{\cos w} + \frac{\cos w}{\cos w}} = \text{RHS}$

59. $\text{LHS} = \dfrac{\sin^2 t + 2\sin t\cos t + \cos^2 t}{\sin t\cos t}$
$= \dfrac{\sin^2 t + \cos^2 t}{\sin t\cos t} + \dfrac{2\sin t\cos t}{\sin t\cos t} = \dfrac{1}{\sin t\cos t} + 2$
$= \text{RHS}$

60. $\text{LHS} = \dfrac{1}{\cos t}\cdot\dfrac{1}{\sin t}\cdot\left(\dfrac{\sin t}{\cos t}+\dfrac{\cos t}{\sin t}\right)$

$= \dfrac{1}{\cos^2 t}+\dfrac{1}{\sin^2 t} = \text{RHS}$

61. $\text{LHS} = \dfrac{1+\frac{\sin^2 u}{\cos^2 u}}{1-\frac{\sin^2 u}{\cos^2 u}}\cdot\dfrac{\cos^2 u}{\cos^2 u} = \dfrac{\cos^2 u+\sin^2 u}{\cos^2 u-\sin^2 u} = \text{RHS}$

62. $\text{LHS} = \dfrac{1+\sec^2 x}{\sec^2 x} = \dfrac{1}{\sec^2 x}+1 = \cos^2 x+1 = \text{RHS}$

63. $\text{LHS} = \dfrac{\sec x}{\sec x-\tan x}\cdot\dfrac{\sec x+\tan x}{\sec x+\tan x}$

$= \dfrac{\sec x(\sec x+\tan x)}{\sec^2 x-\tan^2 x} = \text{RHS}$

64. $\text{LHS} = \dfrac{\frac{1}{\cos x}+\frac{1}{\sin x}}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}\cdot\dfrac{\sin x\cos x}{\sin x\cos x} = \dfrac{\sin x+\cos x}{\sin^2 x+\cos^2 x} = \text{RHS}$

65. $\text{LHS} = (\sec v-\tan v)\cdot\dfrac{\sec v+\tan v}{\sec v+\tan v}$

$= \dfrac{\sec^2 v-\tan^2 v}{\sec v+\tan v} = \text{RHS}$

66. $\text{LHS} = \dfrac{\sin A}{1-\cos A}\cdot\dfrac{1+\cos A}{1+\cos A}-\cot A$

$= \dfrac{\sin A(1+\cos A)}{1-\cos^2 A}-\cot A$

$= \dfrac{\sin A(1+\cos A)}{\sin^2 A}-\dfrac{\cos A}{\sin A}$

$= \dfrac{1}{\sin A}+\dfrac{\cos A}{\sin A}-\dfrac{\cos A}{\sin A} = \text{RHS}$

67. $\text{LHS} = \dfrac{\sin x+\cos x}{\frac{1}{\cos x}+\frac{1}{\sin x}} = \dfrac{\sin x+\cos x}{\frac{\sin x+\cos x}{\cos x\sin x}}$

$= (\sin x+\cos x)\dfrac{\cos x\sin x}{\sin x+\cos x} = \text{RHS}$

68. $\text{LHS} = \dfrac{1-\cos x}{\sin x}\cdot\dfrac{1-\cos x}{1-\cos x}+\dfrac{\sin x}{1-\cos x}\cdot\dfrac{\sin x}{\sin x}$

$= \dfrac{1-2\cos x+\cos^2 x+\sin^2 x}{\sin x(1-\cos x)} = \dfrac{2-2\cos x}{\sin x(1-\cos x)}$

$= \dfrac{2(1-\cos x)}{\sin x(1-\cos x)} = \text{RHS}$

69. $\text{LHS} = \dfrac{\frac{1}{\sin x}-\frac{\cos x}{\sin x}}{\frac{1}{\cos x}-1}\cdot\dfrac{\sin x\cos x}{\sin x\cos x} = \dfrac{\cos x(1-\cos x)}{\sin x(1-\cos x)}$

$= \dfrac{\cos x}{\sin x} = \text{RHS}$

70. $\text{LHS} = \dfrac{1}{\sec^2 x} = \cos^2 x = \text{RHS}$

71. $\text{LHS} = \dfrac{\sin^2 u}{\cos^2 u}-\dfrac{\sin^2 u\cos^2 u}{\cos^2 u} = \dfrac{\sin^2 u}{\cos^2 u}(1-\cos^2 u) = \text{RHS}$

72. $\text{LHS} = \dfrac{\tan v\sin v}{\tan v+\sin v}\cdot\dfrac{\tan v-\sin v}{\tan v-\sin v}$

$= \dfrac{\tan v\sin v(\tan v-\sin v)}{\tan^2 v-\sin^2 v}$

$= \dfrac{\tan v\sin v(\tan v-\sin v)}{\sin^2 v(\sec^2 v-1)}$

$= \dfrac{\tan v\sin v(\tan v-\sin v)}{\sin^2 v\tan^2 v} = \text{RHS}$

73. $\text{LHS} = (\sec^2 x-\tan^2 x)(\sec^2 x+\tan^2 x) = \text{RHS}$

74. $\text{LHS} = \dfrac{\cos\theta}{1-\sin\theta}\cdot\dfrac{1+\sin\theta}{1+\sin\theta} = \dfrac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$

$= \dfrac{\cos\theta(1+\sin\theta)}{\cos^2\theta} = \text{RHS}$

75. $\text{RHS} = \dfrac{\sin\theta-\frac{1}{\sin\theta}}{\cos\theta-\frac{\cos\theta}{\sin\theta}} = \dfrac{\frac{\sin^2\theta-1}{\sin\theta}}{\frac{\cos\theta\sin\theta-\cos\theta}{\sin\theta}}$

$= \dfrac{\cos^2\theta}{\cos\theta(\sin\theta-1)} = \text{LHS}$

76. $\text{LHS} = \dfrac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}\cdot\dfrac{\cos x}{\cos x} = \dfrac{\cos x+\sin x}{\cos x-\sin x} = \text{RHS}$

77. $\text{LHS} = \dfrac{-\sin^2 t+\tan^2 t}{\sin^2 t} = -1+\dfrac{\sin^2 t}{\cos^2 t}\cdot\dfrac{1}{\sin^2 t}$

$= -1+\sec^2 t = \text{RHS}$

78. $\text{LHS} = \dfrac{(1+\sin x)-(1-\sin x)}{(1-\sin x)(1+\sin x)} = \dfrac{2\sin x}{1-\sin^2 x} = \dfrac{2\sin x}{\cos^2 x}$

$= 2\dfrac{1}{\cos x}\cdot\dfrac{\sin x}{\cos x} = \text{RHS}$

79. $\text{LHS} = \dfrac{\sec x-\tan x+\sec x+\tan x}{(\sec x+\tan x)(\sec x-\tan x)}$

$= \dfrac{2\sec x}{\sec^2 x-\tan^2 x} = \text{RHS}$

80. $\text{LHS} = \dfrac{(1+\sin x)^2-(1-\sin x)^2}{(1-\sin x)(1+\sin x)}$

$= \dfrac{1+2\sin x+\sin^2 x-1+2\sin x-\sin^2 x}{1-\sin^2 x}$

$= \dfrac{4\sin x}{\cos^2 x} = 4\dfrac{\sin x}{\cos x}\cdot\dfrac{1}{\cos x} = \text{RHS}$

81. $\text{LHS} = \tan^2 x+2\tan x\cot x+\cot^2 x = \tan^2 x+2+\cot^2 x$

$= (\tan^2 x+1)+(\cot^2 x+1) = \text{RHS}$

82. $\text{LHS} = (\sec^2 x-1)-(\csc^2 x-1)$

$= \sec^2 x-\csc^2 x = \text{RHS}$

83. $\text{LHS} = \dfrac{\frac{1}{\cos u}-1}{\frac{1}{\cos u}+1}\cdot\dfrac{\cos u}{\cos u} = \text{RHS}$

84. $\text{LHS} = \dfrac{\cot x+1}{\cot x-1}\cdot\dfrac{\tan x}{\tan x} = \dfrac{\tan x\cot x+\tan x}{\tan x\cot x-\tan x}$

$= \dfrac{1+\tan x}{1-\tan x} = \text{RHS}$

85. $\text{LHS} = \dfrac{(\sin x+\cos x)(\sin^2 x-\sin x\cos x+\cos^2 x)}{\sin x+\cos x}$

$= \sin^2 x-\sin x\cos x+\cos^2 x = \text{RHS}$

86. $\text{LHS} = \dfrac{\tan v-\cot v}{(\tan v-\cot v)(\tan v+\cot v)}$

$= \dfrac{1}{\tan v+\cot v} = \dfrac{1}{\frac{\sin v}{\cos v}+\frac{\cos v}{\sin v}}\cdot\dfrac{\sin v\cos v}{\sin v\cos v}$

$= \dfrac{\sin v\cos v}{\sin^2 v+\cos^2 v} = \text{RHS}$

87. $\text{LHS} = \dfrac{1+\sin x}{1-\sin x}\cdot\dfrac{1+\sin x}{1+\sin x} = \dfrac{(1+\sin x)^2}{1-\sin^2 x}$

$= \dfrac{(1+\sin x)^2}{\cos^2 x} = \left(\dfrac{1+\sin x}{\cos x}\right)^2 = \text{RHS}$

88. $\text{LHS} = \dfrac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{\frac{\cos x}{\sin x} + \frac{\cos y}{\sin y}} = \left(\dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y}\right)$

$\times \left(\dfrac{\sin x \sin y}{\cos x \sin y + \sin x \cos y}\right)$

$= \dfrac{\sin x \sin y}{\cos x \cos y} = \tan x \tan y = \text{RHS}$

89. $\text{LHS} = \left(\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}\right)^4 = \left(\dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right)^4$

$= \left(\dfrac{1}{\sin x \cos x}\right)^4 = \text{RHS}$

90. $\text{LHS} = \left(\sin\alpha - \dfrac{\sin\alpha}{\cos\alpha}\right)\left(\cos\alpha - \dfrac{\cos\alpha}{\sin\alpha}\right)$

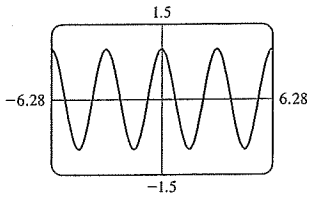
$= \sin\alpha\left(1 - \dfrac{1}{\cos\alpha}\right)\cdot\cos\alpha\left(1 - \dfrac{1}{\sin\alpha}\right)$

$= \cos\alpha\left(1 - \dfrac{1}{\cos\alpha}\right)\sin\alpha\left(1 - \dfrac{1}{\sin\alpha}\right)$

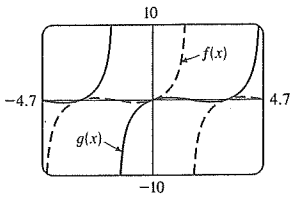
$= (\cos\alpha - 1)(\sin\alpha - 1) = \text{RHS}$

91. $\tan\theta$ **92.** $\sec\theta$ **93.** $\tan\theta$ **94.** $\frac{1}{8}\cot^2\theta\cos\theta$
95. $3\cos\theta$ **96.** $\sin\theta$

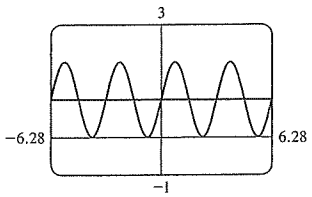
97. Yes



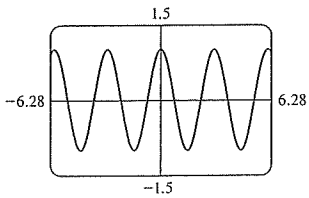
98. No



99. No



100. Yes



SECTION 7.2 ■ PAGE 505

1. addition; $\sin x \cos y + \cos x \sin y$
2. subtraction; $\cos x \cos y + \sin x \sin y$

3. $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ **4.** $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ **5.** $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

6. $-\dfrac{\sqrt{2}+\sqrt{6}}{4}$ **7.** $2-\sqrt{3}$ **8.** $\sqrt{3}-2$

9. $-\dfrac{\sqrt{6}+\sqrt{2}}{4}$ **10.** $\dfrac{\sqrt{2}-\sqrt{6}}{4}$ **11.** $\sqrt{3}-2$

12. $-\dfrac{\sqrt{6}+\sqrt{2}}{4}$ **13.** $-\dfrac{\sqrt{6}+\sqrt{2}}{4}$ **14.** $-(2+\sqrt{3})$

15. $\sqrt{2}/2$ **16.** 0 **17.** $\frac{1}{2}$ **18.** $\sqrt{3}/3$ **19.** $\sqrt{3}$ **20.** $-\frac{1}{2}$

21. $\text{LHS} = \dfrac{\sin(\frac{\pi}{2}-u)}{\cos(\frac{\pi}{2}-u)} = \dfrac{\sin\frac{\pi}{2}\cos u - \cos\frac{\pi}{2}\sin u}{\cos\frac{\pi}{2}\cos u + \sin\frac{\pi}{2}\sin u}$

$= \dfrac{\cos u}{\sin u} = \text{RHS}$

22. $\text{LHS} = \dfrac{\cos(\frac{\pi}{2}-u)}{\sin(\frac{\pi}{2}-u)} = \dfrac{\cos\frac{\pi}{2}\cos u + \sin\frac{\pi}{2}\sin u}{\sin\frac{\pi}{2}\cos u - \cos\frac{\pi}{2}\sin u}$

$= \dfrac{\sin u}{\cos u} = \text{RHS}$

23. $\text{LHS} = \dfrac{1}{\cos(\frac{\pi}{2}-u)} = \dfrac{1}{\cos\frac{\pi}{2}\cos u + \sin\frac{\pi}{2}\sin u}$

$= \dfrac{1}{\sin u} = \text{RHS}$

24. $\text{LHS} = \dfrac{1}{\sin(\frac{\pi}{2}-u)} = \dfrac{1}{\sin\frac{\pi}{2}\cos u - \cos\frac{\pi}{2}\sin u}$

$= \dfrac{1}{\cos u} = \text{RHS}$

25. $\text{LHS} = \sin x\cos\frac{\pi}{2} - \cos x\sin\frac{\pi}{2} = \text{RHS}$
26. $\text{LHS} = \cos x\cos\frac{\pi}{2} + \sin x\sin\frac{\pi}{2} = \text{RHS}$
27. $\text{LHS} = \sin x\cos\pi - \cos x\sin\pi = \text{RHS}$
28. $\text{LHS} = \cos x\cos\pi + \sin x\sin\pi = \text{RHS}$

29. $\text{LHS} = \dfrac{\tan x - \tan\pi}{1 + \tan x\tan\pi} = \text{RHS}$

30. $\text{LHS} = \sin\left(\dfrac{\pi}{2} - x\right) = \sin\dfrac{\pi}{2}\cos x - \cos\dfrac{\pi}{2}\sin x = \cos x$

$\text{RHS} = \sin\left(\dfrac{\pi}{2} + x\right) = \sin\dfrac{\pi}{2}\cos x + \cos\dfrac{\pi}{2}\sin x = \cos x$

31. $\text{LHS} = \cos x\cos\frac{\pi}{6} - \sin x\sin\frac{\pi}{6} + \sin x\cos\frac{\pi}{3} - \cos x\sin\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x + \frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x = \text{RHS}$

32. $\text{LHS} = \dfrac{\tan x - \tan\frac{\pi}{4}}{1 + \tan x\tan\frac{\pi}{4}} = \text{RHS}$

33. $\text{LHS} = \sin x\cos y + \cos x\sin y$

$-(\sin x\cos y - \cos x\sin y) = \text{RHS}$

34. $\text{LHS} = \cos x\cos y - \sin x\sin y + \cos x\cos y$

$+ \sin x\sin y = \text{RHS}$

35. $\text{LHS} = \dfrac{1}{\tan(x-y)} = \dfrac{1 + \tan x\tan y}{\tan x - \tan y}$

$= \dfrac{1 + \frac{1}{\cot x}\frac{1}{\cot y}}{\frac{1}{\cot x} - \frac{1}{\cot y}}\cdot\dfrac{\cot x\cot y}{\cot x\cot y} = \text{RHS}$

36. $\text{LHS} = \dfrac{1}{\tan(x+y)} = \dfrac{1 - \tan x\tan y}{\tan x + \tan y}$

$= \dfrac{1 - \frac{1}{\cot x}\frac{1}{\cot y}}{\frac{1}{\cot x} + \frac{1}{\cot y}}\cdot\dfrac{\cot x\cot y}{\cot x\cot y}$

$= \dfrac{\cot x\cot y - 1}{\cot x + \cot y} = \text{RHS}$