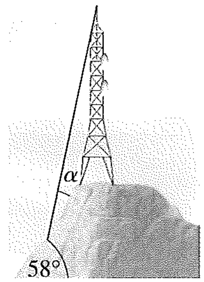
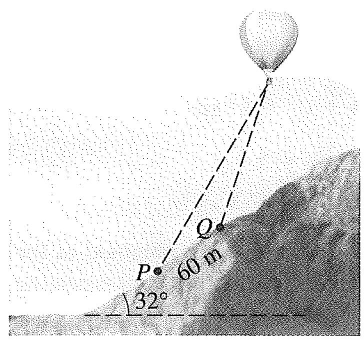
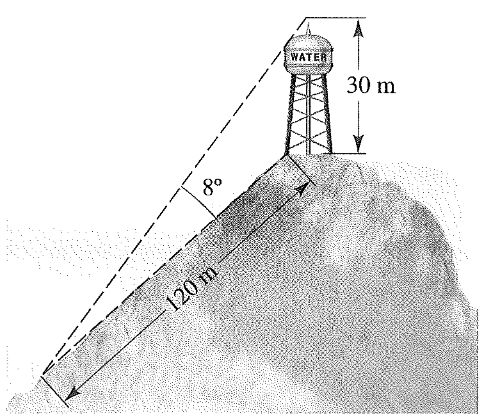
40. Length of a Guy Wire A communications tower is located at the top of a steep hill, as shown. The angle of inclination of the hill is 58°. A guy wire is to be attached to the top of the tower and to the ground, 100 m downhill from the base of the tower. The angle α in the figure is determined to be 12°. Find the length of cable required for the guy wire.



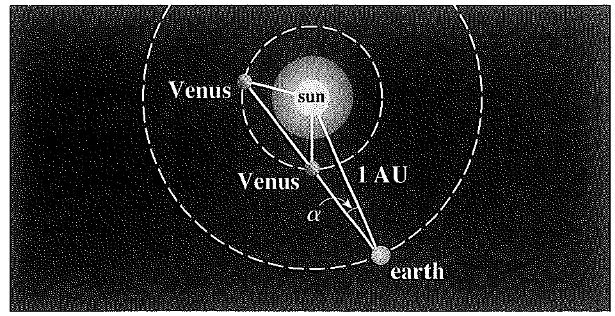
41. Calculating a Distance Observers at P and Q are located on the side of a hill that is inclined 32° to the horizontal, as shown. The observer at P determines the angle of elevation to a hot-air balloon to be 62°. At the same instant the observer at Q measures the angle of elevation to the balloon to be 71°. If P is 60 m down the hill from Q, find the distance from Q to the balloon.



42. Calculating an Angle A water tower 30 m tall is located at the top of a hill. From a distance of 120 m down the hill, it is observed that the angle formed between the top and base of the tower is 8°. Find the angle of inclination of the hill.



43. Distances to Venus The *elongation* α of a planet is the angle formed by the planet, earth, and sun (see the figure). It is known that the distance from the sun to Venus is 0.723 AU (see Exercise 65 in Section 6.2). At a certain time the elongation of Venus is found to be 39.4°. Find the possible distances from the earth to Venus at that time in astronomical units (AU).



44. Soap Bubbles When two bubbles cling together in midair, their common surface is part of a sphere whose center D lies on the line passing throught the centers of the bubbles (see the figure). Also, angles ACB and ACD each have measure 60°.

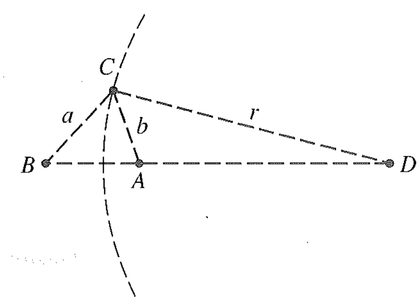
(a) Show that the radius r of the common face is given by

$$r = \frac{ab}{a - b}$$

[*Hint:* Use the Law of Sines together with the fact that an angle θ and its supplement $180° - \theta$ have the same sine.]

(b) Find the radius of the common face if the radii of the bubbles are 4 cm and 3 cm.

(c) What shape does the common face take if the two bubbles have equal radii?



DISCOVERY ■ DISCUSSION ■ WRITING

45. Number of Solutions in the Ambiguous Case We have seen that when the Law of Sines is used to solve a triangle in the SSA case, there may be two, one, or no solution(s). Sketch triangles like those in Figure 6 to verify the criteria in the table for the number of solutions if you are given $\angle A$ and sides a and b.

Criterion	Number of solutions
$a \geq b$	1
$b > a > b \sin A$	2
$a = b \sin A$	1
$a < b \sin A$	0

If $\angle A = 30°$ and $b = 100$, use these criteria to find the range of values of a for which the triangle ABC has two solutions, one solution, or no solution.

6.6 THE LAW OF COSINES

| The Law of Cosines ▶ Navigation: Heading and Bearing ▶ The Area of a Triangle

The Law of Cosines

The Law of Sines cannot be used directly to solve triangles if we know two sides and the angle between them or if we know all three sides (these are Cases 3 and 4 of the preceding section). In these two cases the **Law of Cosines** applies.

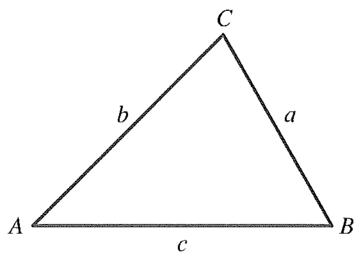


FIGURE 1

THE LAW OF COSINES

In any triangle ABC (see Figure 1), we have

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

PROOF To prove the Law of Cosines, place triangle ABC so that $\angle A$ is at the origin, as shown in Figure 2. The coordinates of vertices B and C are $(c, 0)$ and $(b\cos A, b\sin A)$, respectively. (You should check that the coordinates of these points will be the same if we draw angle A as an acute angle.) Using the Distance Formula, we get

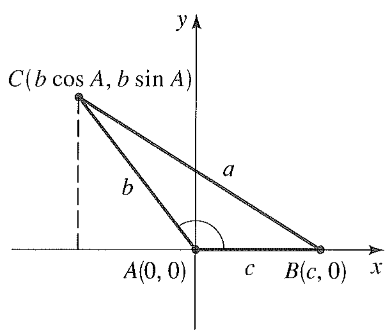


FIGURE 2

$$\begin{aligned} a^2 &= (b\cos A - c)^2 + (b\sin A - 0)^2 \\ &= b^2\cos^2 A - 2bc\cos A + c^2 + b^2\sin^2 A \\ &= b^2(\cos^2 A + \sin^2 A) - 2bc\cos A + c^2 \\ &= b^2 + c^2 - 2bc\cos A \qquad \text{Because } \sin^2 A + \cos^2 A = 1 \end{aligned}$$

This proves the first formula. The other two formulas are obtained in the same way by placing each of the other vertices of the triangle at the origin and repeating the preceding argument. ■

In words, the Law of Cosines says that the square of any side of a triangle is equal to the sum of the squares of the other two sides, minus twice the product of those two sides times the cosine of the included angle.

If one of the angles of a triangle, say $\angle C$, is a right angle, then $\cos C = 0$, and the Law of Cosines reduces to the Pythagorean Theorem, $c^2 = a^2 + b^2$. Thus the Pythagorean Theorem is a special case of the Law of Cosines.

EXAMPLE 1 | Length of a Tunnel

A tunnel is to be built through a mountain. To estimate the length of the tunnel, a surveyor makes the measurements shown in Figure 3. Use the surveyor's data to approximate the length of the tunnel.

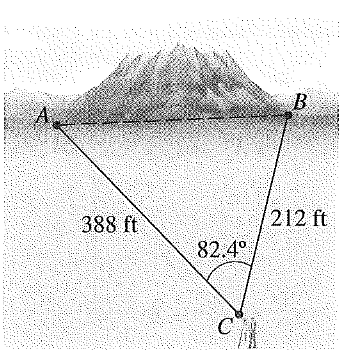


FIGURE 3

SOLUTION To approximate the length c of the tunnel, we use the Law of Cosines:

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos C && \text{Law of Cosines} \\ &= 388^2 + 212^2 - 2(388)(212)\cos 82.4° && \text{Substitute} \\ &\approx 173730.2367 && \text{Use a calculator} \\ c &\approx \sqrt{173730.2367} \approx 416.8 && \text{Take square roots} \end{aligned}$$

Thus the tunnel will be approximately 417 ft long.

NOW TRY EXERCISES **3** AND **39** ■

EXAMPLE 2 | SSS, the Law of Cosines

The sides of a triangle are $a = 5$, $b = 8$, and $c = 12$ (see Figure 4). Find the angles of the triangle.

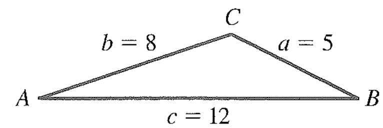


FIGURE 4

SOLUTION We first find $\angle A$. From the Law of Cosines, $a^2 = b^2 + c^2 - 2bc \cos A$. Solving for $\cos A$, we get

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{8^2 + 12^2 - 5^2}{2(8)(12)} = \frac{183}{192} = 0.953125$$

COS⁻¹ OR INV COS OR ARC COS

Using a calculator, we find that $\angle A = \cos^{-1}(0.953125) \approx 18°$. In the same way we get

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{5^2 + 12^2 - 8^2}{2(5)(12)} = 0.875$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{5^2 + 8^2 - 12^2}{2(5)(8)} = -0.6875$$

Using a calculator, we find that

$$\angle B = \cos^{-1}(0.875) \approx 29° \qquad \text{and} \qquad \angle C = \cos^{-1}(-0.6875) \approx 133°$$

Of course, once two angles have been calculated, the third can more easily be found from the fact that the sum of the angles of a triangle is 180°. However, it's a good idea to calculate all three angles using the Law of Cosines and add the three angles as a check on your computations.

NOW TRY EXERCISE **7**

EXAMPLE 3 | SAS, the Law of Cosines

Solve triangle ABC, where $\angle A = 46.5°$, $b = 10.5$, and $c = 18.0$.

SOLUTION We can find a using the Law of Cosines.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \\ &= (10.5)^2 + (18.0)^2 - 2(10.5)(18.0)(\cos 46.5°) \approx 174.05 \end{aligned}$$

Thus, $a \approx \sqrt{174.05} \approx 13.2$. We also use the Law of Cosines to find $\angle B$ and $\angle C$, as in Example 2.

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{13.2^2 + 18.0^2 - 10.5^2}{2(13.2)(18.0)} \approx 0.816477$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{13.2^2 + 10.5^2 - 18.0^2}{2(13.2)(10.5)} \approx -0.142532$$

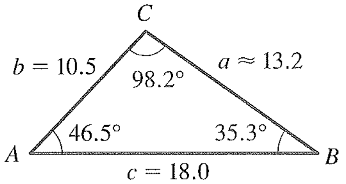


FIGURE 5

Using a calculator, we find that

$$\angle B = \cos^{-1}(0.816477) \approx 35.3° \qquad \text{and} \qquad \angle C = \cos^{-1}(-0.142532) \approx 98.2°$$

To summarize: $\angle B \approx 35.3°$, $\angle C \approx 98.2°$, and $a \approx 13.2$. (See Figure 5.)

NOW TRY EXERCISE **13**

We could have used the Law of Sines to find $\angle B$ and $\angle C$ in Example 3, since we knew all three sides and an angle in the triangle. But knowing the sine of an angle does not uniquely specify the angle, since an angle θ and its supplement $180° - \theta$ both have the same sine. Thus, we would need to decide which of the two angles is the correct choice. This ambiguity does not arise when we use the Law of Cosines, because every angle between 0° and 180° has a unique cosine. So using only the Law of Cosines is preferable in problems like Example 3.

Navigation: Heading and Bearing

In navigation a direction is often given as a **bearing**, that is, as an acute angle measured from due north or due south. The bearing N 30° E, for example, indicates a direction that points 30° to the east of due north (see Figure 6).

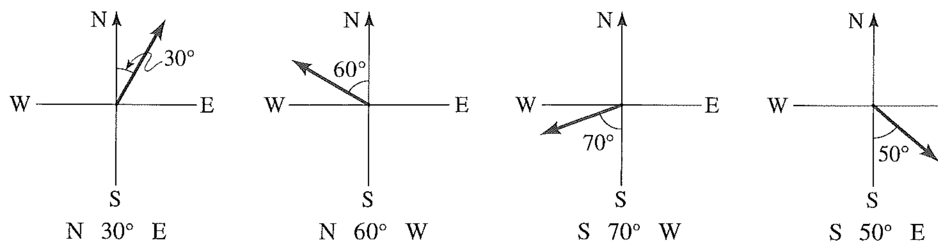


FIGURE 6

EXAMPLE 4 | Navigation

A pilot sets out from an airport and heads in the direction N 20° E, flying at 200 mi/h. After one hour, he makes a course correction and heads in the direction N 40° E. Half an hour after that, engine trouble forces him to make an emergency landing.

(a) Find the distance between the airport and his final landing point.

(b) Find the bearing from the airport to his final landing point.

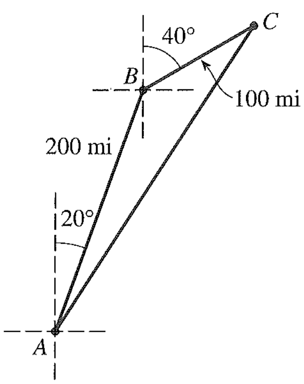


FIGURE 7

SOLUTION

(a) In one hour the plane travels 200 mi, and in half an hour it travels 100 mi, so we can plot the pilot's course as in Figure 7. When he makes his course correction, he turns 20° to the right, so the angle between the two legs of his trip is $180° - 20° = 160°$. So by the Law of Cosines we have

$$b^2 = 200^2 + 100^2 - 2 \cdot 200 \cdot 100 \cos 160°$$
$$\approx 87{,}587.70$$

Thus, $b \approx 295.95$. The pilot lands about 296 mi from his starting point.

(b) We first use the Law of Sines to find $\angle A$.

$$\frac{\sin A}{100} = \frac{\sin 160°}{295.95}$$

$$\sin A = 100 \cdot \frac{\sin 160°}{295.95}$$

$$\approx 0.11557$$

Another angle with sine 0.11557 is $180° - 6.636° = 173.364°$. But this is clearly too large to be $\angle A$ in $\angle ABC$.

Using the $\boxed{\text{SIN}^{-1}}$ key on a calculator, we find that $\angle A \approx 6.636°$. From Figure 7 we see that the line from the airport to the final landing site points in the direction $20° + 6.636° = 26.636°$ east of due north. Thus, the bearing is about N 26.6° E.

NOW TRY EXERCISE 45

The Area of a Triangle

An interesting application of the Law of Cosines involves a formula for finding the area of a triangle from the lengths of its three sides (see Figure 8).

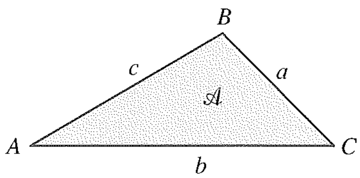


FIGURE 8

HERON'S FORMULA

The area $\mathcal{A}$ of triangle ABC is given by

$$\mathcal{A} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{2}(a + b + c)$ is the **semiperimeter** of the triangle; that is, s is half the perimeter.

PROOF We start with the formula $\mathcal{A} = \frac{1}{2}ab \sin C$ from Section 6.3. Thus

$$\begin{aligned} \mathcal{A}^2 &= \tfrac{1}{4}a^2b^2 \sin^2 C \\ &= \tfrac{1}{4}a^2b^2(1 - \cos^2 C) && \text{Pythagorean identity} \\ &= \tfrac{1}{4}a^2b^2(1 - \cos C)(1 + \cos C) && \text{Factor} \end{aligned}$$

Next, we write the expressions $1 - \cos C$ and $1 + \cos C$ in terms of a, b, and c. By the Law of Cosines we have

$$\begin{aligned} \cos C &= \frac{a^2 + b^2 - c^2}{2ab} && \text{Law of Cosines} \\ 1 + \cos C &= 1 + \frac{a^2 + b^2 - c^2}{2ab} && \text{Add 1} \\ &= \frac{2ab + a^2 + b^2 - c^2}{2ab} && \text{Common denominator} \\ &= \frac{(a + b)^2 - c^2}{2ab} && \text{Factor} \\ &= \frac{(a + b + c)(a + b - c)}{2ab} && \text{Difference of squares} \end{aligned}$$

Similarly

$$1 - \cos C = \frac{(c + a - b)(c - a + b)}{2ab}$$

Substituting these expressions in the formula we obtained for $\mathcal{A}^2$ gives

$$\begin{aligned} \mathcal{A}^2 &= \tfrac{1}{4}a^2b^2 \frac{(a + b + c)(a + b - c)}{2ab} \frac{(c + a - b)(c - a + b)}{2ab} \\ &= \frac{(a + b + c)}{2} \frac{(a + b - c)}{2} \frac{(c + a - b)}{2} \frac{(c - a + b)}{2} \\ &= s(s - c)(s - b)(s - a) \end{aligned}$$

To see that the factors in the last two products are equal, note for example that

$$\frac{a + b - c}{2} = \frac{a + b + c}{2} - c = s - c$$

Heron's Formula now follows by taking the square root of each side. ■

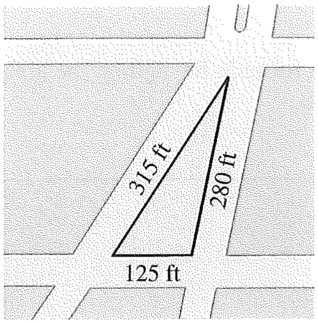


FIGURE 9

EXAMPLE 5 | Area of a Lot

A businessman wishes to buy a triangular lot in a busy downtown location (see Figure 9). The lot frontages on the three adjacent streets are 125, 280, and 315 ft. Find the area of the lot.

SOLUTION The semiperimeter of the lot is

$$s = \frac{125 + 280 + 315}{2} = 360$$

By Heron's Formula the area is

$$\mathscr{A} = \sqrt{360(360 - 125)(360 - 280)(360 - 315)} \approx 17{,}451.6$$

Thus, the area is approximately 17,452 ft^2.

NOW TRY EXERCISES **29** AND **53**

6.6 EXERCISES

CONCEPTS

1. For triangle ABC with sides a, b, and c the Law of Cosines states

$c^2 =$ ______________________

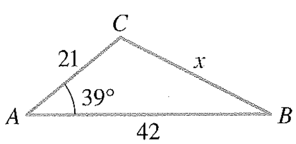
2. In which of the following cases must the Law of Cosines be used to solve a triangle?

ASA SSS SAS SSA

SKILLS

3–10 ■ Use the Law of Cosines to determine the indicated side x or angle θ.

3.



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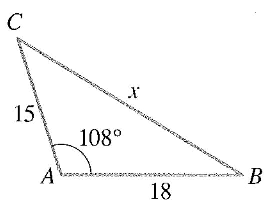

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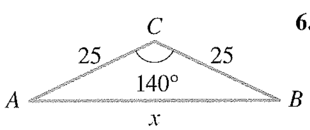

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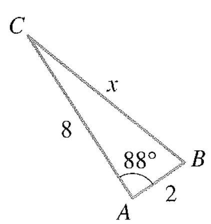

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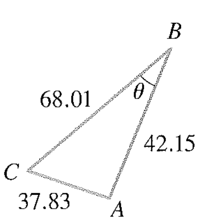

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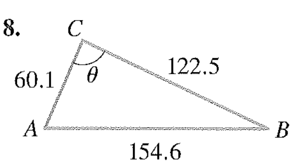

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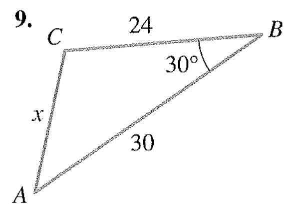

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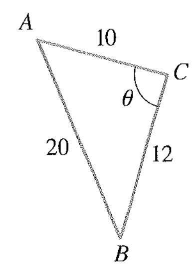
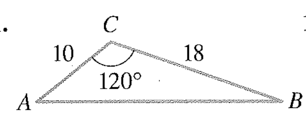


11–20 ■ Solve triangle ABC.

11.



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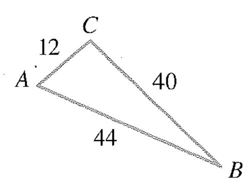


13. $a = 3.0, \quad b = 4.0, \quad \angle C = 53°$

14. $b = 60, \quad c = 30, \quad \angle A = 70°$

15. $a = 20, \quad b = 25, \quad c = 22$

16. $a = 10, \quad b = 12, \quad c = 16$

17. $b = 125, \quad c = 162, \quad \angle B = 40°$

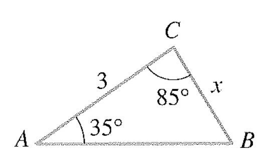
18. $a = 65, \quad c = 50, \quad \angle C = 52°$

19. $a = 50, \quad b = 65, \quad \angle A = 55°$

20. $a = 73.5, \quad \angle B = 61°, \quad \angle C = 83°$

21–28 ■ Find the indicated side x or angle θ. (Use either the Law of Sines or the Law of Cosines, as appropriate.)

21.



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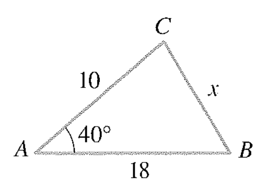

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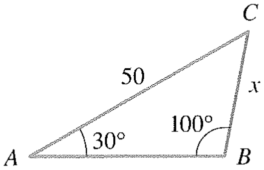

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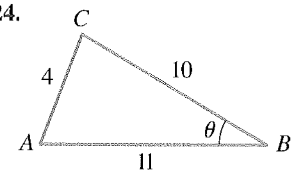

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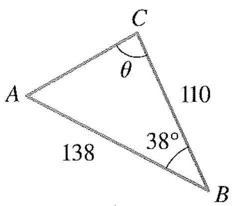

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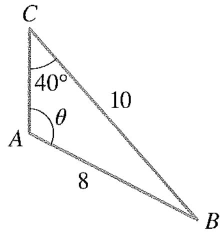

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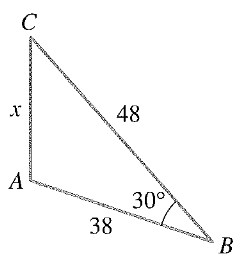

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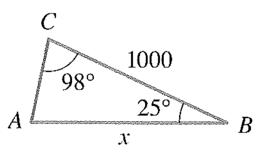


29–32 ■ Find the area of the triangle whose sides have the given lengths.

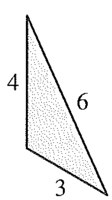
29. $a = 9, \quad b = 12, \quad c = 15$ **30.** $a = 1, \quad b = 2, \quad c = 2$

31. $a = 7, \quad b = 8, \quad c = 9$

32. $a = 11, \quad b = 100, \quad c = 101$

33–36 ■ Find the area of the shaded figure, rounded to two decimals.

33.



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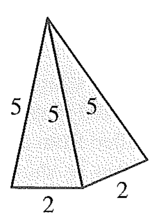

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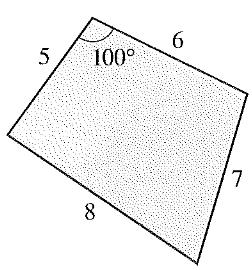
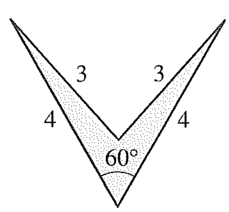
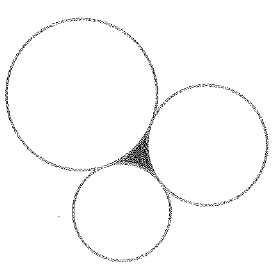


36.



37. Three circles of radii 4, 5, and 6 cm are mutually tangent. Find the shaded area enclosed between the circles.



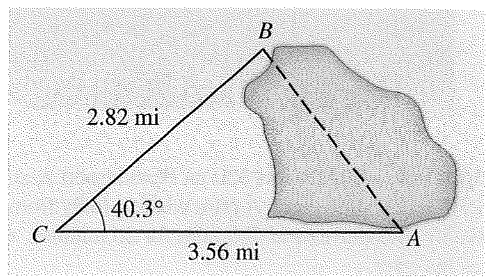
38. Prove that in triangle ABC

$$a = b \cos C + c \cos B$$
$$b = c \cos A + a \cos C$$
$$c = a \cos B + b \cos A$$

These are called the *Projection Laws*. [*Hint:* To get the first equation, add the second and third equations in the Law of Cosines and solve for a.]

APPLICATIONS

39. Surveying To find the distance across a small lake, a surveyor has taken the measurements shown. Find the distance across the lake using this information.



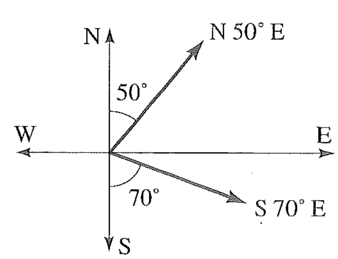
40. Geometry A parallelogram has sides of lengths 3 and 5, and one angle is 50°. Find the lengths of the diagonals.

41. Calculating Distance Two straight roads diverge at an angle of 65°. Two cars leave the intersection at 2:00 P.M., one traveling at 50 mi/h and the other at 30 mi/h. How far apart are the cars at 2:30 P.M.?

42. Calculating Distance A car travels along a straight road, heading east for 1 h, then traveling for 30 min on another road that leads northeast. If the car has maintained a constant speed of 40 mi/h, how far is it from its starting position?

43. Dead Reckoning A pilot flies in a straight path for 1 h 30 min. She then makes a course correction, heading 10° to the right of her original course, and flies 2 h in the new direction. If she maintains a constant speed of 625 mi/h, how far is she from her starting position?

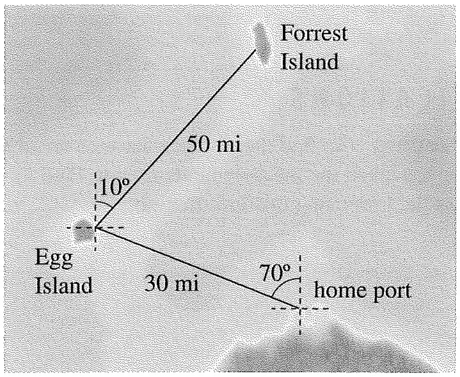
44. Navigation Two boats leave the same port at the same time. One travels at a speed of 30 mi/h in the direction N 50° E and the other travels at a speed of 26 mi/h in a direction S 70° E (see the figure). How far apart are the two boats after one hour?



45. Navigation A fisherman leaves his home port and heads in the direction N 70° W. He travels 30 mi and reaches Egg Island. The next day he sails N 10° E for 50 mi, reaching Forrest Island.

(a) Find the distance between the fisherman's home port and Forrest Island.

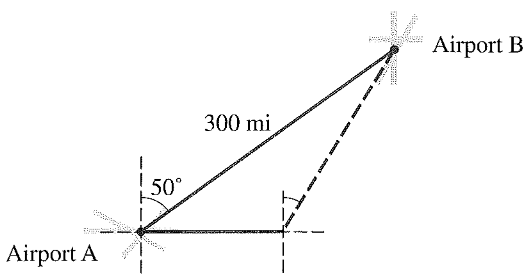
(b) Find the bearing from Forrest Island back to his home port.



46. Navigation Airport B is 300 mi from airport A at a bearing N 50° E (see the figure). A pilot wishing to fly from A to B mistakenly flies due east at 200 mi/h for 30 minutes, when he notices his error.

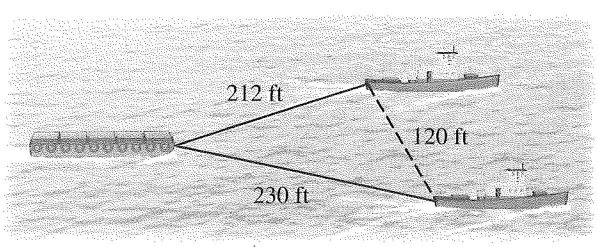
(a) How far is the pilot from his destination at the time he notices the error?

(b) What bearing should he head his plane in order to arrive at airport B?

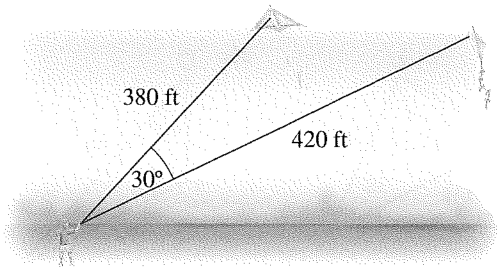


47. Triangular Field A triangular field has sides of lengths 22, 36, and 44 yd. Find the largest angle.

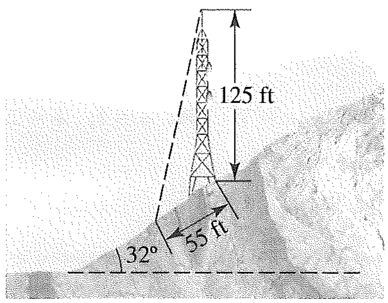
48. Towing a Barge Two tugboats that are 120 ft apart pull a barge, as shown. If the length of one cable is 212 ft and the length of the other is 230 ft, find the angle formed by the two cables.



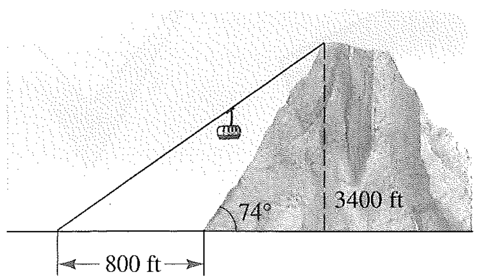
49. Flying Kites A boy is flying two kites at the same time. He has 380 ft of line out to one kite and 420 ft to the other. He estimates the angle between the two lines to be 30°. Approximate the distance between the kites.



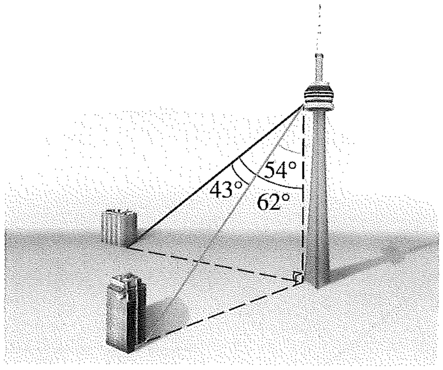
50. Securing a Tower A 125-ft tower is located on the side of a mountain that is inclined 32° to the horizontal. A guy wire is to be attached to the top of the tower and anchored at a point 55 ft downhill from the base of the tower. Find the shortest length of wire needed.



51. Cable Car A steep mountain is inclined 74° to the horizontal and rises 3400 ft above the surrounding plain. A cable car is to be installed from a point 800 ft from the base to the top of the mountain, as shown. Find the shortest length of cable needed.



52. CN Tower The CN Tower in Toronto, Canada, is the tallest free-standing structure in North America. A woman on the observation deck, 1150 ft above the ground, wants to determine the distance between two landmarks on the ground below. She observes that the angle formed by the lines of sight to these two landmarks is 43°. She also observes that the angle between the vertical and the line of sight to one of the landmarks is 62° and to the other landmark is 54°. Find the distance between the two landmarks.



53. Land Value Land in downtown Columbia is valued at \$20 a square foot. What is the value of a triangular lot with sides of lengths 112, 148, and 190 ft?

DISCOVERY ■ DISCUSSION ■ WRITING

54. Solving for the Angles in a Triangle The paragraph that follows the solution of Example 3 on page 477 explains an alternative method for finding $\angle B$ and $\angle C$, using the Law of Sines. Use this method to solve the triangle in the example, finding $\angle B$ first and then $\angle C$. Explain how you chose the appropriate value for the measure of $\angle B$. Which method do you prefer for solving an SAS triangle problem, the one explained in Example 3 or the one you used in this exercise?

CHAPTER 6 | REVIEW

CONCEPT CHECK

1. (a) Explain the difference between a positive angle and a negative angle.
   (b) How is an angle of measure 1 degree formed?
   (c) How is an angle of measure 1 radian formed?
   (d) How is the radian measure of an angle θ defined?
   (e) How do you convert from degrees to radians?
   (f) How do you convert from radians to degrees?
2. (a) When is an angle in standard position?
   (b) When are two angles coterminal?
3. (a) What is the length s of an arc of a circle with radius r that subtends a central angle of θ radians?
   (b) What is the area A of a sector of a circle with radius r and central angle θ radians?
4. If θ is an acute angle in a right triangle, define the six trigonometric ratios in terms of the adjacent and opposite sides and the hypotenuse.
5. What does it mean to solve a triangle?
6. If θ is an angle in standard position, $P(x, y)$ is a point on the terminal side, and r is the distance from the origin to P, write expressions for the six trigonometric functions of θ.
7. Which trigonometric functions are positive in Quadrants I, II, III, and IV?
8. If θ is an angle in standard position, what is its reference angle $\bar{\theta}$?
9. (a) State the reciprocal identities.
   (b) State the Pythagorean identities.
10. (a) What is the area of a triangle with sides of length a and b and with included angle θ?
    (b) What is the area of a triangle with sides of length a, b, and c?
11. Define the inverse sine function $\sin^{-1}$. What are its domain and range?
12. Define the inverse cosine function $\cos^{-1}$. What are its domain and range?
13. Define the inverse tangent function $\tan^{-1}$. What are its domain and range?
14. (a) State the Law of Sines.
    (b) State the Law of Cosines.
15. Explain the ambiguous case in the Law of Sines.

EXERCISES

1–2 ■ Find the radian measure that corresponds to the given degree measure.

1. (a) 60° (b) 330° (c) −135° (d) −90°
2. (a) 24° (b) −330° (c) 750° (d) 5°

3–4 ■ Find the degree measure that corresponds to the given radian measure.

3. (a) $\dfrac{5\pi}{2}$ (b) $-\dfrac{\pi}{6}$ (c) $\dfrac{9\pi}{4}$ (d) 3.1
4. (a) 8 (b) $-\frac{5}{2}$ (c) $\dfrac{11\pi}{6}$ (d) $\dfrac{3\pi}{5}$
5. Find the length of an arc of a circle of radius 8 m if the arc subtends a central angle of 1 rad.
6. Find the measure of a central angle θ in a circle of radius 5 ft if the angle is subtended by an arc of length 7 ft.
7. A circular arc of length 100 ft subtends a central angle of 70°. Find the radius of the circle.
8. How many revolutions will a car wheel of diameter 28 in. make over a period of half an hour if the car is traveling at 60 mi/h?

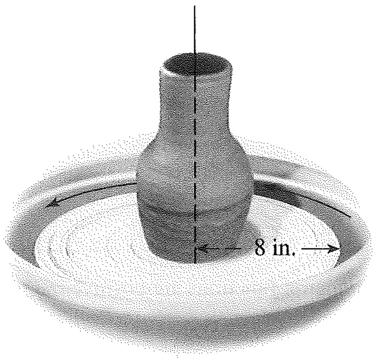
9. New York and Los Angeles are 2450 mi apart. Find the angle that the arc between these two cities subtends at the center of the earth. (The radius of the earth is 3960 mi.)

10. Find the area of a sector with central angle 2 rad in a circle of radius 5 m.

11. Find the area of a sector with central angle 52° in a circle of radius 200 ft.

12. A sector in a circle of radius 25 ft has an area of 125 ft^2. Find the central angle of the sector.

13. A potter's wheel with radius 8 in. spins at 150 rpm. Find the angular and linear speeds of a point on the rim of the wheel.



14. In an automobile transmission a *gear ratio* g is the ratio

$$g = \frac{\text{angular speed of engine}}{\text{angular speed of wheels}}$$

The angular speed of the engine is shown on the tachometer (in rpm).

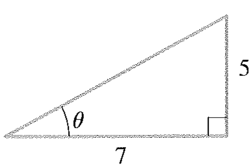
A certain sports car has wheels with radius 11 in. Its gear ratios are shown in the following table. Suppose the car is in fourth gear and the tachometer reads 3500 rpm.

(a) Find the angular speed of the engine.
(b) Find the angular speed of the wheels.
(c) How fast (in mi/h) is the car traveling?

Gear	Ratio
1st	4.1
2nd	3.0
3rd	1.6
4th	0.9
5th	0.7

15–16 ■ Find the values of the six trigonometric ratios of θ.

15.



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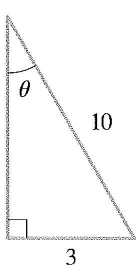
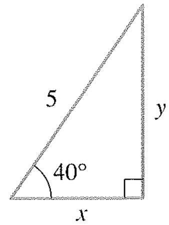


17–20 ■ Find the sides labeled x and y, rounded to two decimal places.

17.



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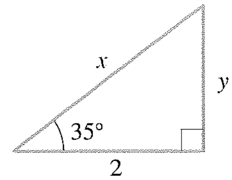

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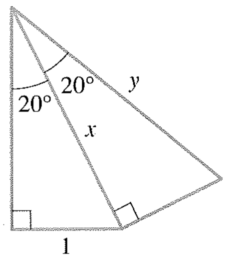

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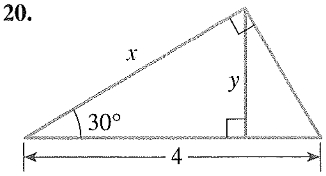
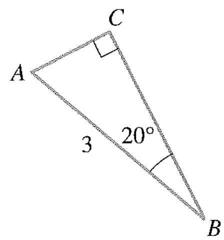


21–24 ■ Solve the triangle.

21.



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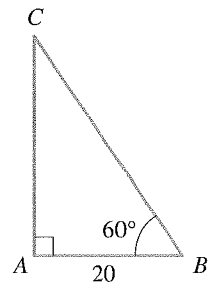

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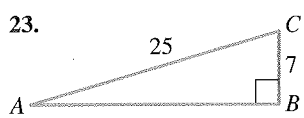

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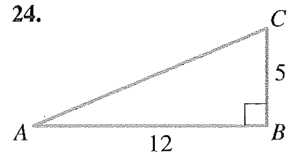
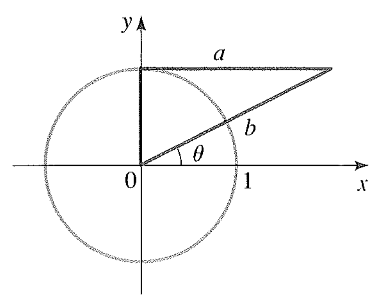


25. Express the lengths a and b in the figure in terms of the trigonometric ratios of θ.



26. The highest free-standing tower in North America is the CN Tower in Toronto, Canada. From a distance of 1 km from its base, the angle of elevation to the top of the tower is 28.81°. Find the height of the tower.

27. Find the perimeter of a regular hexagon that is inscribed in a circle of radius 8 m.