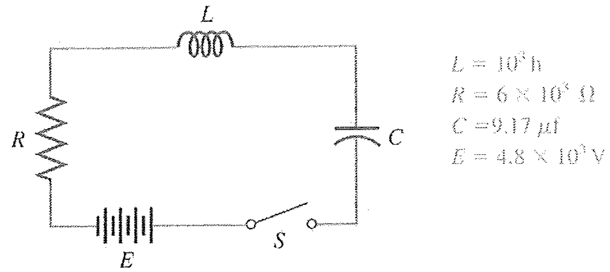
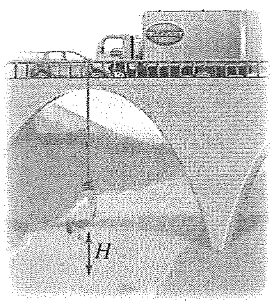
**83. Electric Circuit** After the switch is closed in the circuit shown, the current $t$ seconds later is $I(t) = 0.8e^{-3t}\sin 10t$. Find the current at the times
(a) $t = 0.1$ s and (b) $t = 0.5$ s.




**84. Bungee Jumping** A bungee jumper plummets from a high bridge to the river below and then bounces back over and over again. At time $t$ seconds after her jump, her height $H$ (in meters) above the river is given by $H(t) = 100 + 75e^{-t/20}\cos(\frac{\pi}{4}t)$. Find her height at the times indicated in the table.

| $t$ | $H(t)$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 4 | |
| 6 | |
| 8 | |
| 12 | |

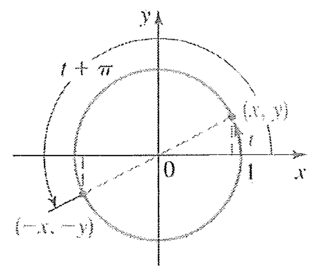



## DISCOVERY ▪ DISCUSSION ▪ WRITING

**85. Reduction Formulas** A *reduction formula* is one that can be used to "reduce" the number of terms in the input for a trigonometric function. Explain how the figure shows that the following reduction formulas are valid:

$$\sin(t + \pi) = -\sin t \qquad \cos(t + \pi) = -\cos t$$

$$\tan(t + \pi) = \tan t$$

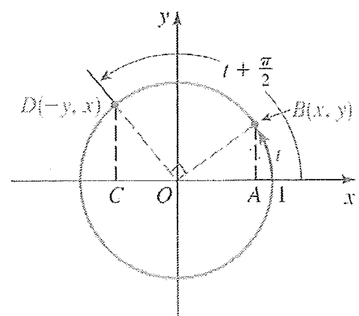



**86. More Reduction Formulas** By the "Angle-Side-Angle" theorem from elementary geometry, triangles $CDO$ and $AOB$ in the figure are congruent. Explain how this proves that if $B$ has coordinates $(x, y)$, then $D$ has coordinates $(-y, x)$. Then explain how the figure shows that the following reduction formulas are valid:

$$\sin\left(t + \frac{\pi}{2}\right) = \cos t$$

$$\cos\left(t + \frac{\pi}{2}\right) = -\sin t$$

$$\tan\left(t + \frac{\pi}{2}\right) = -\cot t$$




# 5.3 Trigonometric Graphs

Graphs of Sine and Cosine ▶ Graphs of Transformations of Sine and Cosine ▶ Using Graphing Devices to Graph Trigonometric Functions

The graph of a function gives us a better idea of its behavior. So, in this section we graph the sine and cosine functions and certain transformations of these functions. The other trigonometric functions are graphed in the next section.

## ▼ Graphs of Sine and Cosine

To help us graph the sine and cosine functions, we first observe that these functions repeat their values in a regular fashion. To see exactly how this happens, recall that the circumference of the unit circle is $2\pi$. It follows that the terminal point $P(x, y)$ determined by the real number $t$ is the same as that determined by $t + 2\pi$. Since the sine and cosine functions are defined in terms of the coordinates of $P(x, y)$, it follows that their values are unchanged by the addition of any integer multiple of $2\pi$. In other words,

$$\sin(t + 2n\pi) = \sin t \qquad \text{for any integer } n$$

$$\cos(t + 2n\pi) = \cos t \qquad \text{for any integer } n$$

Thus, the sine and cosine functions are *periodic* according to the following definition: A function $f$ is **periodic** if there is a positive number $p$ such that $f(t + p) = f(t)$ for every $t$. The least such positive number (if it exists) is the **period** of $f$. If $f$ has period $p$, then the graph of $f$ on any interval of length $p$ is called **one complete period** of $f$.

**PERIODIC PROPERTIES OF SINE AND COSINE**

The functions sine and cosine have period $2\pi$:

$$\sin(t + 2\pi) = \sin t \qquad\qquad \cos(t + 2\pi) = \cos t$$

**TABLE 1**

| $t$ | $\sin t$ | $\cos t$ |
|---|---|---|
| $0 \to \frac{\pi}{2}$ | $0 \to 1$ | $1 \to 0$ |
| $\frac{\pi}{2} \to \pi$ | $1 \to 0$ | $0 \to -1$ |
| $\pi \to \frac{3\pi}{2}$ | $0 \to -1$ | $-1 \to 0$ |
| $\frac{3\pi}{2} \to 2\pi$ | $-1 \to 0$ | $0 \to 1$ |

So the sine and cosine functions repeat their values in any interval of length $2\pi$. To sketch their graphs, we first graph one period. To sketch the graphs on the interval $0 \le t \le 2\pi$, we could try to make a table of values and use those points to draw the graph. Since no such table can be complete, let's look more closely at the definitions of these functions.

Recall that $\sin t$ is the $y$-coordinate of the terminal point $P(x, y)$ on the unit circle determined by the real number $t$. How does the $y$-coordinate of this point vary as $t$ increases? It's easy to see that the $y$-coordinate of $P(x, y)$ increases to 1, then decreases to $-1$ repeatedly as the point $P(x, y)$ travels around the unit circle. (See Figure 1.) In fact, as $t$ increases from 0 to $\pi/2$, $y = \sin t$ increases from 0 to 1. As $t$ increases from $\pi/2$ to $\pi$, the value of $y = \sin t$ decreases from 1 to 0. Table 1 shows the variation of the sine and cosine functions for $t$ between 0 and $2\pi$.

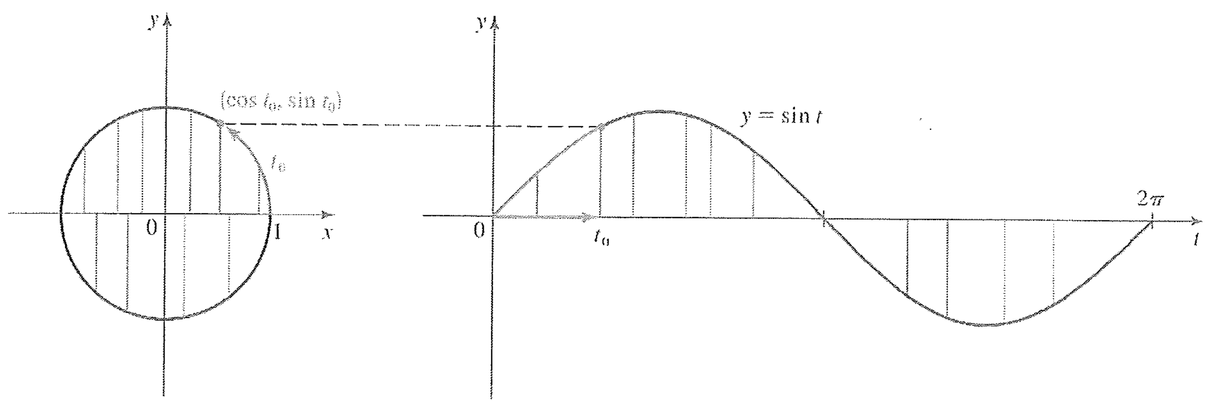



**FIGURE 1**

To draw the graphs more accurately, we find a few other values of $\sin t$ and $\cos t$ in Table 2. We could find still other values with the aid of a calculator.

**TABLE 2**

| $t$ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{5\pi}{6}$ | $\pi$ | $\frac{7\pi}{6}$ | $\frac{4\pi}{3}$ | $\frac{3\pi}{2}$ | $\frac{5\pi}{3}$ | $\frac{11\pi}{6}$ | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sin t$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-\frac{\sqrt{3}}{2}$ | $-1$ | $-\frac{\sqrt{3}}{2}$ | $-\frac{1}{2}$ | 0 |
| $\cos t$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-\frac{\sqrt{3}}{2}$ | $-1$ | $-\frac{\sqrt{3}}{2}$ | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |

Now we use this information to graph the functions sin $t$ and cos $t$ for $t$ between 0 and $2\pi$ in Figures 2 and 3. These are the graphs of one period. Using the fact that these functions are periodic with period $2\pi$, we get their complete graphs by continuing the same pattern to the left and to the right in every successive interval of length $2\pi$.

The graph of the sine function is symmetric with respect to the origin. This is as expected, since sine is an odd function. Since the cosine function is an even function, its graph is symmetric with respect to the $y$-axis.

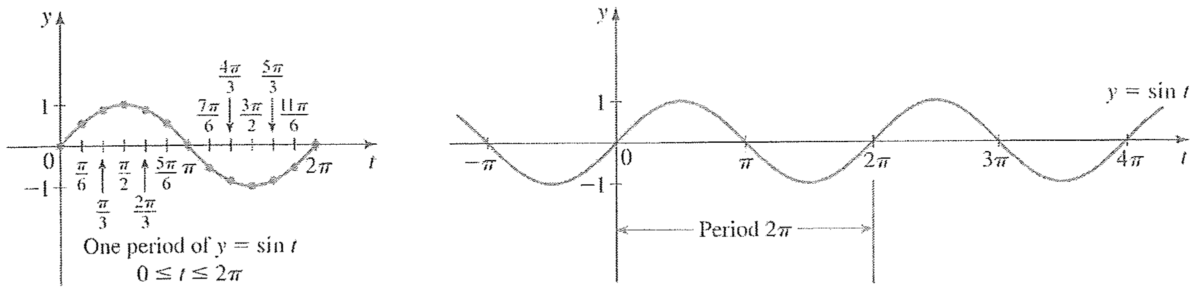



**FIGURE 2** Graph of sin $t$

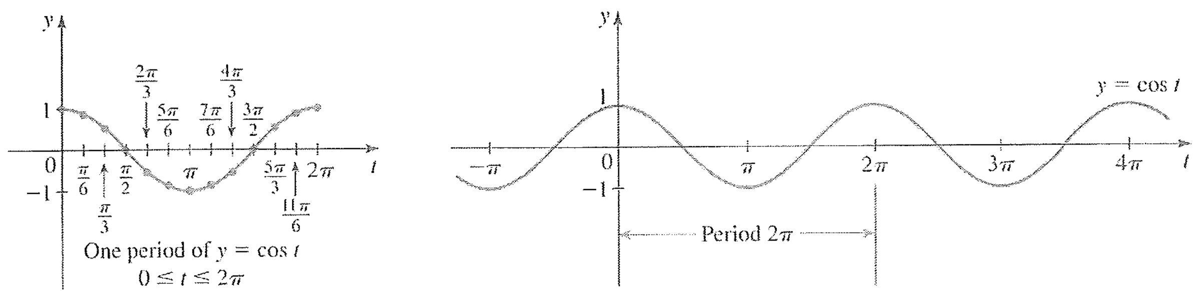



**FIGURE 3** Graph of cos $t$

## Graphs of Transformations of Sine and Cosine

We now consider graphs of functions that are transformations of the sine and cosine functions. Thus, the graphing techniques of Section 2.5 are very useful here. The graphs we obtain are important for understanding applications to physical situations such as harmonic motion (see Section 5.6), but some of them are beautiful graphs that are interesting in their own right.

It's traditional to use the letter $x$ to denote the variable in the domain of a function. So from here on we use the letter $x$ and write $y = \sin x$, $y = \cos x$, $y = \tan x$, and so on to denote these functions.

### EXAMPLE 1 | Cosine Curves

Sketch the graph of each function.

(a) $f(x) = 2 + \cos x$ (b) $g(x) = -\cos x$

**SOLUTION**

(a) The graph of $y = 2 + \cos x$ is the same as the graph of $y = \cos x$, but shifted up 2 units (see Figure 4(a)).

(b) The graph of $y = -\cos x$ in Figure 4(b) is the reflection of the graph of $y = \cos x$ in the $x$-axis.

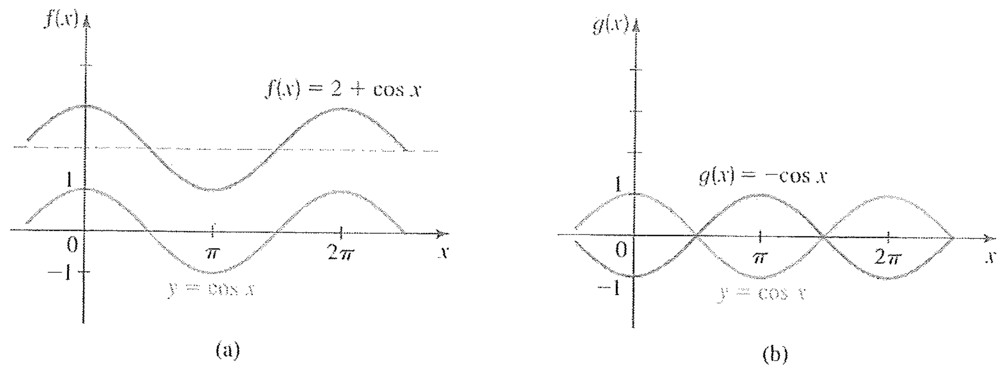



FIGURE 4

NOW TRY EXERCISES 3 AND 5

Let's graph $y = 2\sin x$. We start with the graph of $y = \sin x$ and multiply the $y$-coordinate of each point by 2. This has the effect of stretching the graph vertically by a factor of 2. To graph $y = \frac{1}{2}\sin x$, we start with the graph of $y = \sin x$ and multiply the $y$-coordinate of each point by $\frac{1}{2}$. This has the effect of shrinking the graph vertically by a factor of $\frac{1}{2}$ (see Figure 5).

Vertical stretching and shrinking of graphs is discussed in Section 2.5.

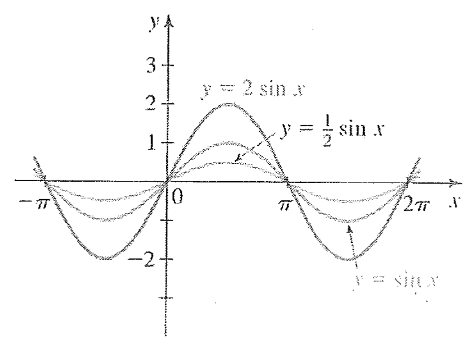



FIGURE 5

In general, for the functions

$$y = a\sin x \qquad \text{and} \qquad y = a\cos x$$

the number $|a|$ is called the **amplitude** and is the largest value these functions attain. Graphs of $y = a\sin x$ for several values of $a$ are shown in Figure 6.

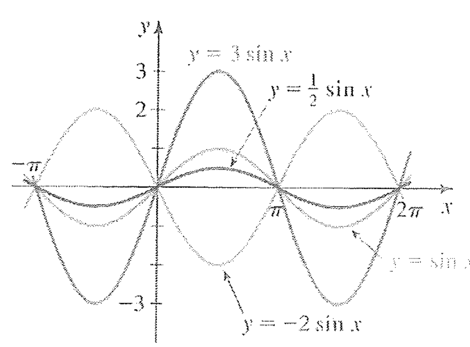



FIGURE 6

## EXAMPLE 2 | Stretching a Cosine Curve

Find the amplitude of $y = -3 \cos x$, and sketch its graph.

**SOLUTION** The amplitude is $|-3| = 3$, so the largest value the graph attains is 3 and the smallest value is $-3$. To sketch the graph, we begin with the graph of $y = \cos x$, stretch the graph vertically by a factor of 3, and reflect in the $x$-axis, arriving at the graph in Figure 7.

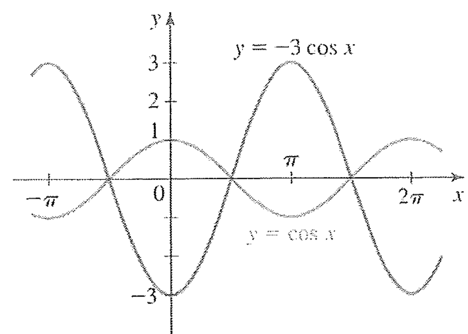



FIGURE 7

NOW TRY EXERCISE 9

Since the sine and cosine functions have period $2\pi$, the functions

$$y = a \sin kx \qquad \text{and} \qquad y = a \cos kx \qquad (k > 0)$$

complete one period as $kx$ varies from 0 to $2\pi$, that is, for $0 \le kx \le 2\pi$ or for $0 \le x \le 2\pi/k$. So these functions complete one period as $x$ varies between 0 and $2\pi/k$ and thus have period $2\pi/k$. The graphs of these functions are called **sine curves** and **cosine curves**, respectively. (Collectively, sine and cosine curves are often referred to as **sinusoidal** curves.)

### SINE AND COSINE CURVES

The sine and cosine curves

$$y = a \sin kx \qquad \text{and} \qquad y = a \cos kx \qquad (k > 0)$$

have **amplitude** $|a|$ and **period** $2\pi/k$.

An appropriate interval on which to graph one complete period is $[0, 2\pi/k]$.

Horizontal stretching and shrinking of graphs is discussed in Section 2.5.

To see how the value of $k$ affects the graph of $y = \sin kx$, let's graph the sine curve $y = \sin 2x$. Since the period is $2\pi/2 = \pi$, the graph completes one period in the interval $0 \le x \le \pi$ (see Figure 8(a)). For the sine curve $y = \sin \frac{1}{2}x$, the period is $2\pi \div \frac{1}{2} = 4\pi$, so the graph completes one period in the interval $0 \le x \le 4\pi$ (see Figure 8(b)). We see that the effect is to *shrink* the graph horizontally if $k > 1$ or to *stretch* the graph horizontally if $k < 1$.

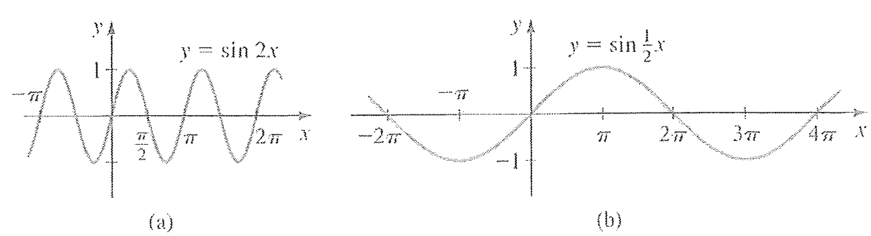



FIGURE 8

For comparison, in Figure 9 we show the graphs of one period of the sine curve $y = a \sin kx$ for several values of $k$.

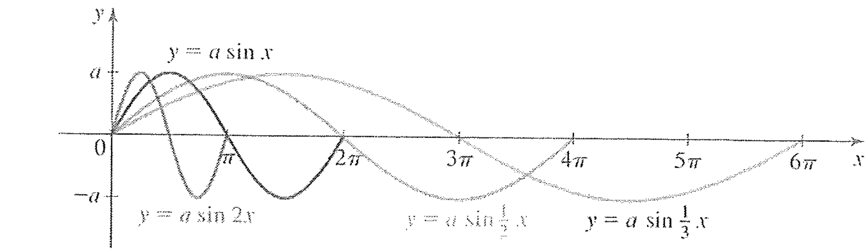



**FIGURE 9**

## EXAMPLE 3 | Amplitude and Period

Find the amplitude and period of each function, and sketch its graph.

**(a)** $y = 4 \cos 3x$ **(b)** $y = -2 \sin \frac{1}{2}x$

**SOLUTION**

**(a)** We get the amplitude and period from the form of the function as follows:

$$\text{amplitude} = |a| = 4$$

$$y = 4 \cos 3x$$

$$\text{period} = \frac{2\pi}{k} = \frac{2\pi}{3}$$

The amplitude is 4 and the period is $2\pi/3$. The graph is shown in Figure 10.

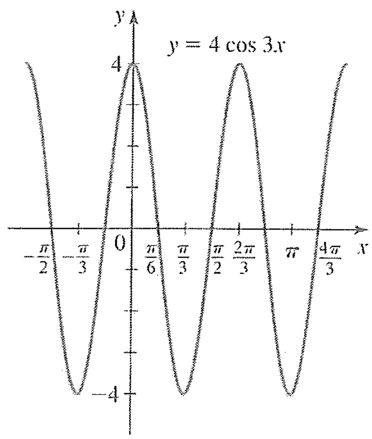



**FIGURE 10**

**(b)** For $y = -2 \sin \frac{1}{2}x$,

$$\text{amplitude} = |a| = |-2| = 2$$

$$\text{period} = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

The graph is shown in Figure 11.

NOW TRY EXERCISES **19** AND **21**

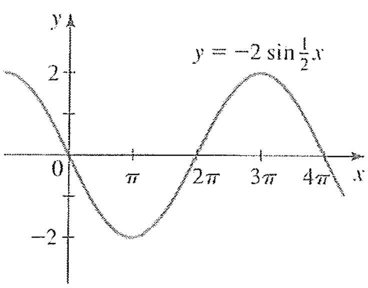



**FIGURE 11**

The graphs of functions of the form $y = a \sin k(x - b)$ and $y = a \cos k(x - b)$ are simply sine and cosine curves shifted horizontally by an amount $|b|$. They are shifted to the right if $b > 0$ or to the left if $b < 0$. The number $b$ is the *phase shift*. We summarize the properties of these functions in the following box.

### SHIFTED SINE AND COSINE CURVES

The sine and cosine curves

$$y = a \sin k(x - b) \quad \text{and} \quad y = a \cos k(x - b) \qquad (k > 0)$$

have **amplitude** $|a|$, **period** $2\pi/k$, and **phase shift** $b$.

An appropriate interval on which to graph one complete period is $[b, b + (2\pi/k)]$.

The graphs of $y = \sin\left(x - \frac{\pi}{3}\right)$ and $y = \sin\left(x + \frac{\pi}{6}\right)$ are shown in Figure 12.

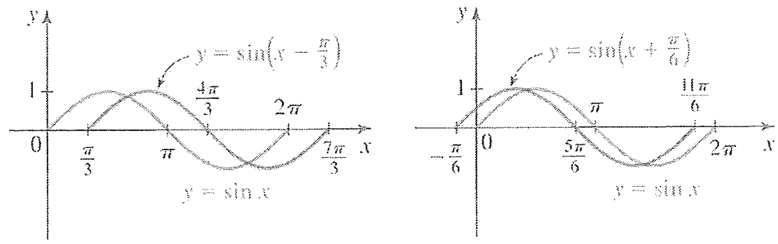



**FIGURE 12**

## EXAMPLE 4 | A Shifted Sine Curve

Find the amplitude, period, and phase shift of $y = 3 \sin 2\left(x - \frac{\pi}{4}\right)$, and graph one complete period.

**SOLUTION** We get the amplitude, period, and phase shift from the form of the function as follows:

$$\text{amplitude} = |a| = 3 \qquad \text{period} = \frac{2\pi}{k} = \frac{2\pi}{2} = \pi$$

$$y = 3 \sin 2\left(x - \frac{\pi}{4}\right)$$

$$\text{phase shift} = \frac{\pi}{4} \text{ (to the right)}$$

Here is another way to find an appropriate interval on which to graph one complete period. Since the period of $y = \sin x$ is $2\pi$, the function $y = 3 \sin 2(x - \frac{\pi}{4})$ will go through one complete period as $2(x - \frac{\pi}{4})$ varies from 0 to $2\pi$.

| Start of period: | End of period: |
|---|---|
| $2(x - \frac{\pi}{4}) = 0$ | $2(x - \frac{\pi}{4}) = 2\pi$ |
| $x - \frac{\pi}{4} = 0$ | $x - \frac{\pi}{4} = \pi$ |
| $x = \frac{\pi}{4}$ | $x = \frac{5\pi}{4}$ |

So we graph one period on the interval $[\frac{\pi}{4}, \frac{5\pi}{4}]$.

Since the phase shift is $\pi/4$ and the period is $\pi$, one complete period occurs on the interval

$$\left[\frac{\pi}{4}, \frac{\pi}{4} + \pi\right] = \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$$

As an aid in sketching the graph, we divide this interval into four equal parts, then graph a sine curve with amplitude 3 as in Figure 13.

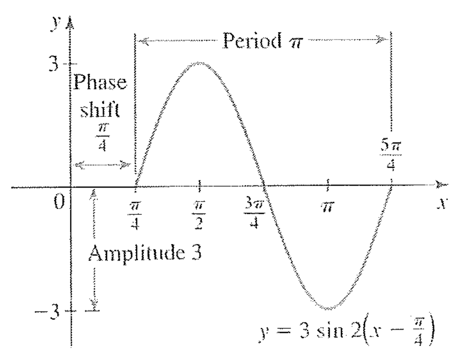



**FIGURE 13**

NOW TRY EXERCISE 33

## EXAMPLE 5 | A Shifted Cosine Curve

Find the amplitude, period, and phase shift of

$$y = \frac{3}{4} \cos\left(2x + \frac{2\pi}{3}\right)$$

and graph one complete period.

**SOLUTION** We first write this function in the form $y = a\cos k(x - b)$. To do this, we factor 2 from the expression $2x + \dfrac{2\pi}{3}$ to get

$$y = \frac{3}{4}\cos 2\left[x - \left(-\frac{\pi}{3}\right)\right]$$

Thus we have

$$\text{amplitude} = |a| = \frac{3}{4}$$

$$\text{period} = \frac{2\pi}{k} = \frac{2\pi}{2} = \pi$$

$$\text{phase shift} = b = -\frac{\pi}{3} \qquad \text{Shift } \frac{\pi}{3} \text{ to the } \textit{left}$$

We can also find one complete period as follows:

| Start of period: | End of period: |
|---|---|
| $2x + \frac{2\pi}{3} = 0$ | $2x + \frac{2\pi}{3} = 2\pi$ |
| $2x = -\frac{2\pi}{3}$ | $2x = \frac{4\pi}{3}$ |
| $x = -\frac{\pi}{3}$ | $x = \frac{2\pi}{3}$ |

So we graph one period on the interval $\left[-\frac{\pi}{3}, \frac{2\pi}{3}\right]$.

From this information it follows that one period of this cosine curve begins at $-\pi/3$ and ends at $(-\pi/3) + \pi = 2\pi/3$. To sketch the graph over the interval $[-\pi/3, 2\pi/3]$, we divide this interval into four equal parts and graph a cosine curve with amplitude $\frac{3}{4}$ as shown in Figure 14.

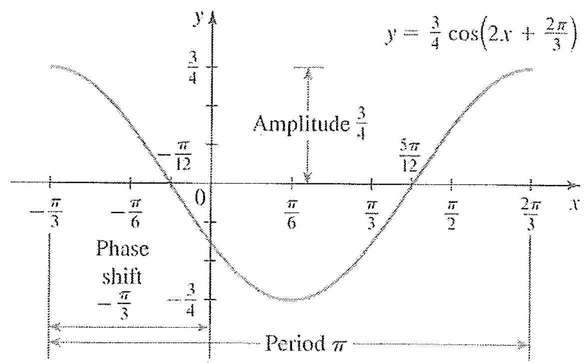



**FIGURE 14**

NOW TRY EXERCISE 35

## Using Graphing Devices to Graph Trigonometric Functions

See Section 1.9 for guidelines on choosing an appropriate viewing rectangle.

When using a graphing calculator or a computer to graph a function, it is important to choose the viewing rectangle carefully in order to produce a reasonable graph of the function. This is especially true for trigonometric functions; the next example shows that, if care is not taken, it's easy to produce a very misleading graph of a trigonometric function.

### EXAMPLE 6 | Choosing the Viewing Rectangle

Graph the function $f(x) = \sin 50x$ in an appropriate viewing rectangle.

The appearance of the graphs in Figure 15 depends on the machine used. The graphs you get with your own graphing device might not look like these figures, but they will also be quite inaccurate.

**SOLUTION** Figure 15(a) shows the graph of $f$ produced by a graphing calculator using the viewing rectangle $[-12, 12]$ by $[-1.5, 1.5]$. At first glance the graph appears to be reasonable. But if we change the viewing rectangle to the ones shown in Figure 15, the graphs look very different. Something strange is happening.

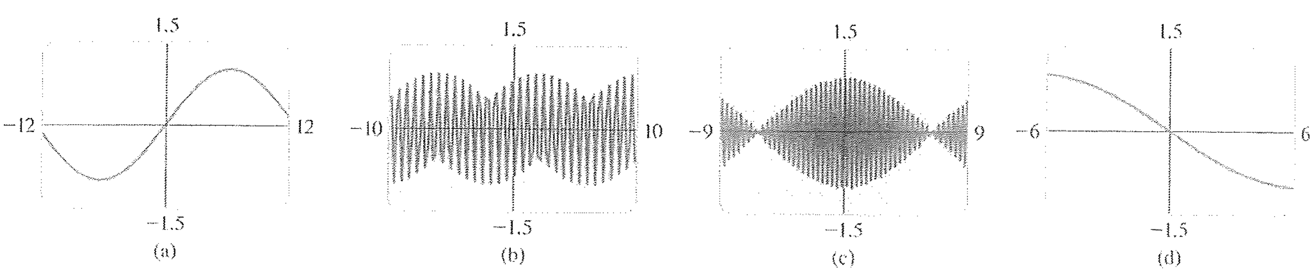



**FIGURE 15** Graphs of $f(x) = \sin 50x$ in different viewing rectangles

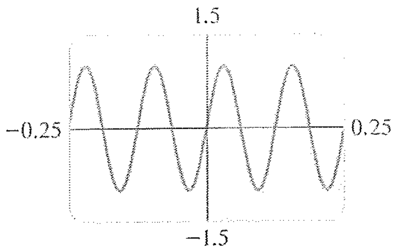



**FIGURE 16** $f(x) = \sin 50x$

To explain the big differences in appearance of these graphs and to find an appropriate viewing rectangle, we need to find the period of the function $y = \sin 50x$:

$$\text{period} = \frac{2\pi}{50} = \frac{\pi}{25} \approx 0.126$$

This suggests that we should deal only with small values of $x$ in order to show just a few oscillations of the graph. If we choose the viewing rectangle $[-0.25, 0.25]$ by $[-1.5, 1.5]$, we get the graph shown in Figure 16.

Now we see what went wrong in Figure 15. The oscillations of $y = \sin 50x$ are so rapid that when the calculator plots points and joins them, it misses most of the maximum and minimum points and therefore gives a very misleading impression of the graph.

NOW TRY EXERCISE 51

The function $h$ in Example 7 is **periodic** with period $2\pi$. In general, functions that are sums of functions from the following list are periodic:

$$1, \cos kx, \cos 2kx, \cos 3kx, \ldots$$

$$\sin kx, \sin 2kx, \sin 3kx, \ldots$$

Although these functions appear to be special, they are actually fundamental to describing all periodic functions that arise in practice. The French mathematician J. B. J. Fourier (see page 501) discovered that nearly every periodic function can be written as a sum (usually an infinite sum) of these functions. This is remarkable because it means that any situation in which periodic variation occurs can be described mathematically using the functions sine and cosine. A modern application of Fourier's discovery is the digital encoding of sound on compact discs.

## EXAMPLE 7 | A Sum of Sine and Cosine Curves

Graph $f(x) = 2\cos x$, $g(x) = \sin 2x$, and $h(x) = 2\cos x + \sin 2x$ on a common screen to illustrate the method of graphical addition.

**SOLUTION** Notice that $h = f + g$, so its graph is obtained by adding the corresponding $y$-coordinates of the graphs of $f$ and $g$. The graphs of $f$, $g$, and $h$ are shown in Figure 17.

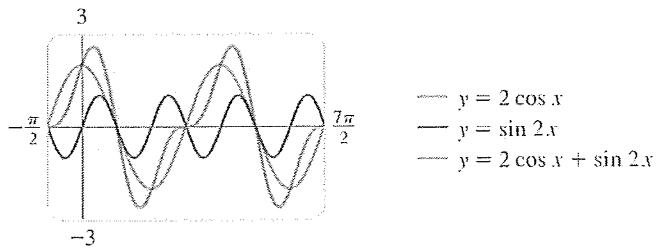



**FIGURE 17**

NOW TRY EXERCISE 59

## EXAMPLE 8 | A Cosine Curve with Variable Amplitude

Graph the functions $y = x^2$, $y = -x^2$, and $y = x^2 \cos 6\pi x$ on a common screen. Comment on and explain the relationship among the graphs.

**SOLUTION** Figure 18 shows all three graphs in the viewing rectangle $[-1.5, 1.5]$ by $[-2, 2]$. It appears that the graph of $y = x^2 \cos 6\pi x$ lies between the graphs of the functions $y = x^2$ and $y = -x^2$.

To understand this, recall that the values of $\cos 6\pi x$ lie between $-1$ and $1$, that is,

$$-1 \le \cos 6\pi x \le 1$$

for all values of $x$. Multiplying the inequalities by $x^2$ and noting that $x^2 \ge 0$, we get

$$-x^2 \le x^2 \cos 6\pi x \le x^2$$

This explains why the functions $y = x^2$ and $y = -x^2$ form a boundary for the graph of $y = x^2 \cos 6\pi x$. (Note that the graphs touch when $\cos 6\pi x = \pm 1$.)

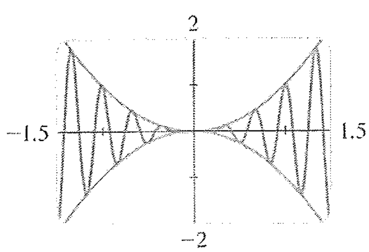
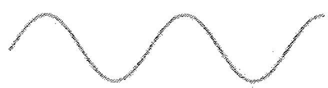


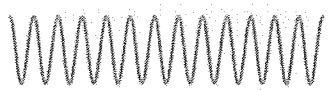

**FIGURE 18** $y = x^2 \cos 6\pi x$

NOW TRY EXERCISE 63

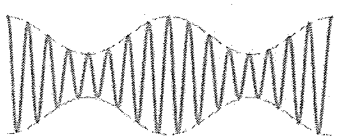
Example 8 shows that the function $y = x^2$ controls the amplitude of the graph of $y = x^2 \cos 6\pi x$. In general, if $f(x) = a(x) \sin kx$ or $f(x) = a(x) \cos kx$, the function $a$ determines how the amplitude of $f$ varies, and the graph of $f$ lies between the graphs of $y = -a(x)$ and $y = a(x)$. Here is another example.

**AM and FM Radio**
Radio transmissions consist of sound waves superimposed on a harmonic electromagnetic wave form called the **carrier signal.**

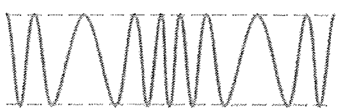

Sound wave


Carrier signal

There are two types of radio transmission, called **amplitude modulation (AM)** and **frequency modulation (FM).** In AM broadcasting, the sound wave changes, or **modulates,** the amplitude of the carrier, but the frequency remains unchanged.


AM signal

In FM broadcasting, the sound wave modulates the frequency, but the amplitude remains the same.


FM signal

## EXAMPLE 9 | A Cosine Curve with Variable Amplitude

Graph the function $f(x) = \cos 2\pi x \cos 16\pi x$.

**SOLUTION** The graph is shown in Figure 19. Although it was drawn by a computer, we could have drawn it by hand, by first sketching the boundary curves $y = \cos 2\pi x$ and $y = -\cos 2\pi x$. The graph of $f$ is a cosine curve that lies between the graphs of these two functions.

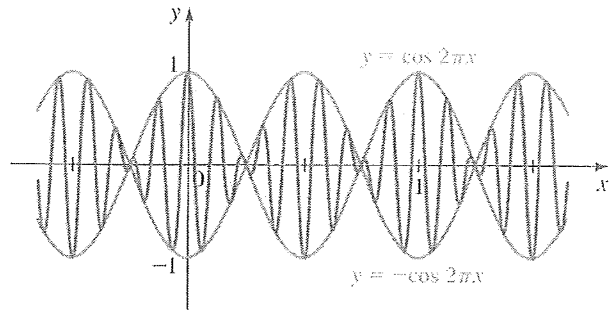



**FIGURE 19** $f(x) = \cos 2\pi x \cos 16\pi x$

NOW TRY EXERCISE 65

## EXAMPLE 10 | A Sine Curve with Decaying Amplitude

The function $f(x) = \dfrac{\sin x}{x}$ is important in calculus. Graph this function and comment on its behavior when $x$ is close to 0.

**SOLUTION** The viewing rectangle $[-15, 15]$ by $[-0.5, 1.5]$ shown in Figure 20(a) gives a good global view of the graph of $f$. The viewing rectangle $[-1, 1]$ by $[-0.5, 1.5]$ in Figure 20(b) focuses on the behavior of $f$ when $x \approx 0$. Notice that although $f(x)$ is not defined when $x = 0$ (in other words, 0 is not in the domain of $f$), the values of $f$ seem to approach 1 when $x$ gets close to 0. This fact is crucial in calculus.

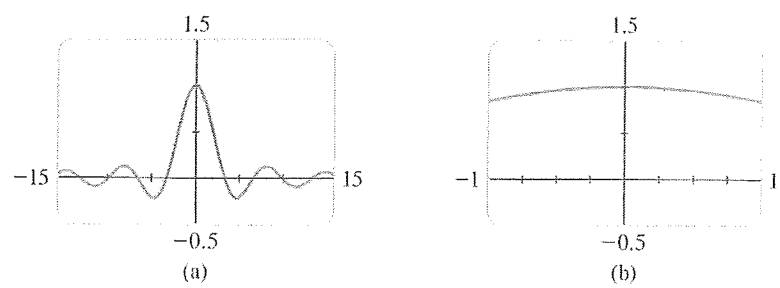



**FIGURE 20** $f(x) = \dfrac{\sin x}{x}$

NOW TRY EXERCISE 75

The function in Example 10 can be written as

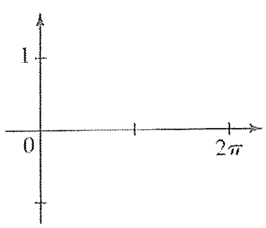
$$f(x) = \frac{1}{x}\sin x$$

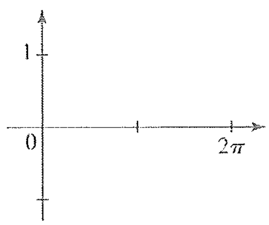
and may thus be viewed as a sine function whose amplitude is controlled by the function $a(x) = 1/x$.

## 5.3 EXERCISES

### CONCEPTS

1. The trigonometric functions $y = \sin x$ and $y = \cos x$ have amplitude ________ and period ________. Sketch a graph of each function on the interval $[0, 2\pi]$.




2. The trigonometric function $y = 3 \sin 2x$ has amplitude ________ and period ________.

### SKILLS

**3–16** ■ Graph the function.

3. $f(x) = 1 + \cos x$
4. $f(x) = 3 + \sin x$
5. $f(x) = -\sin x$
6. $f(x) = 2 - \cos x$
7. $f(x) = -2 + \sin x$
8. $f(x) = -1 + \cos x$
9. $g(x) = 3 \cos x$
10. $g(x) = 2 \sin x$
11. $g(x) = -\frac{1}{2} \sin x$
12. $g(x) = -\frac{2}{3} \cos x$
13. $g(x) = 3 + 3 \cos x$
14. $g(x) = 4 - 2 \sin x$
15. $h(x) = |\cos x|$
16. $h(x) = |\sin x|$

**17–28** ■ Find the amplitude and period of the function, and sketch its graph.

17. $y = \cos 2x$
18. $y = -\sin 2x$
19. $y = -3 \sin 3x$
20. $y = \frac{1}{2} \cos 4x$
21. $y = 10 \sin \frac{1}{2}x$
22. $y = 5 \cos \frac{1}{4}x$
23. $y = -\frac{1}{3} \cos \frac{1}{3}x$
24. $y = 4 \sin(-2x)$
25. $y = -2 \sin 2\pi x$
26. $y = -3 \sin \pi x$
27. $y = 1 + \frac{1}{2} \cos \pi x$
28. $y = -2 + \cos 4\pi x$

**29–42** ■ Find the amplitude, period, and phase shift of the function, and graph one complete period.

29. $y = \cos\left(x - \frac{\pi}{2}\right)$
30. $y = 2 \sin\left(x - \frac{\pi}{3}\right)$
31. $y = -2 \sin\left(x - \frac{\pi}{6}\right)$
32. $y = 3 \cos\left(x + \frac{\pi}{4}\right)$
33. $y = -4 \sin 2\left(x + \frac{\pi}{2}\right)$
34. $y = \sin \frac{1}{2}\left(x + \frac{\pi}{4}\right)$
35. $y = 5 \cos\left(3x - \frac{\pi}{4}\right)$
36. $y = 2 \sin\left(\frac{2}{3}x - \frac{\pi}{6}\right)$
37. $y = \frac{1}{2} - \frac{1}{2} \cos\left(2x - \frac{\pi}{3}\right)$
38. $y = 1 + \cos\left(3x + \frac{\pi}{2}\right)$
39. $y = 3 \cos \pi(x + \frac{1}{2})$
40. $y = 3 + 2 \sin 3(x + 1)$
41. $y = \sin(\pi + 3x)$
42. $y = \cos\left(\frac{\pi}{2} - x\right)$

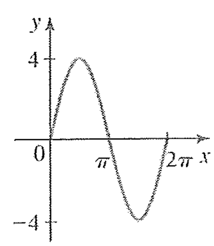
**43–50** ■ The graph of one complete period of a sine or cosine curve is given.

(a) Find the amplitude, period, and phase shift.

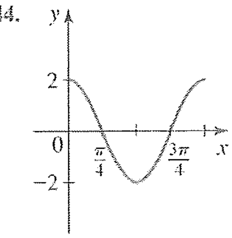
(b) Write an equation that represents the curve in the form

$$y = a \sin k(x - b) \quad \text{or} \quad y = a \cos k(x - b)$$

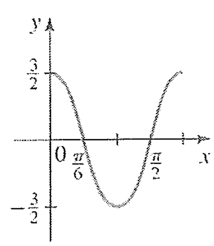
43.



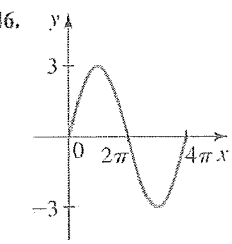

44.



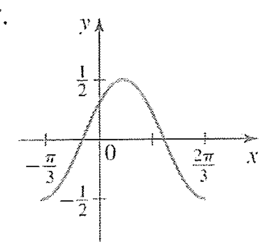

45.



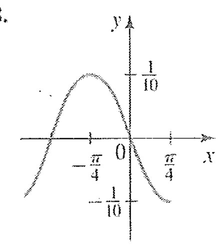

46.



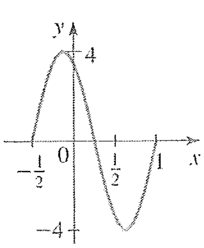

47.



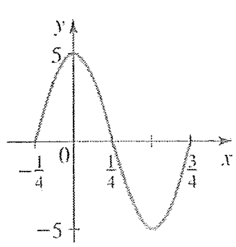

48.




49.




50.




**51–58** ■ Determine an appropriate viewing rectangle for each function, and use it to draw the graph.

51. $f(x) = \cos 100x$
52. $f(x) = 3 \sin 120x$
53. $f(x) = \sin(x/40)$
54. $f(x) = \cos(x/80)$
55. $y = \tan 25x$
56. $y = \csc 40x$
57. $y = \sin^2 20x$
58. $y = \sqrt{\tan 10\pi x}$

**59–60** ■ Graph $f$, $g$, and $f + g$ on a common screen to illustrate graphical addition.

59. $f(x) = x$, $g(x) = \sin x$

60. $f(x) = \sin x$, $g(x) = \sin 2x$

**61–66** ■ Graph the three functions on a common screen. How are the graphs related?

61. $y = x^2$, $y = -x^2$, $y = x^2 \sin x$

62. $y = x$, $y = -x$, $y = x \cos x$

63. $y = \sqrt{x}$, $y = -\sqrt{x}$, $y = \sqrt{x} \sin 5\pi x$

64. $y = \dfrac{1}{1 + x^2}$, $y = -\dfrac{1}{1 + x^2}$, $y = \dfrac{\cos 2\pi x}{1 + x^2}$

65. $y = \cos 3\pi x$, $y = -\cos 3\pi x$, $y = \cos 3\pi x \cos 21\pi x$

66. $y = \sin 2\pi x$, $y = -\sin 2\pi x$, $y = \sin 2\pi x \sin 10\pi x$

**67–70** ■ Find the maximum and minimum values of the function.

67. $y = \sin x + \sin 2x$

68. $y = x - 2 \sin x$, $0 \leq x \leq 2\pi$

69. $y = 2 \sin x + \sin^2 x$

70. $y = \dfrac{\cos x}{2 + \sin x}$

**71–74** ■ Find all solutions of the equation that lie in the interval $[0, \pi]$. State each answer correct to two decimal places.

71. $\cos x = 0.4$

72. $\tan x = 2$

73. $\csc x = 3$

74. $\cos x = x$

**75–76** ■ A function $f$ is given.

(a) Is $f$ even, odd, or neither?

(b) Find the $x$-intercepts of the graph of $f$.

(c) Graph $f$ in an appropriate viewing rectangle.

(d) Describe the behavior of the function as $x \to \pm\infty$.

(e) Notice that $f(x)$ is not defined when $x = 0$. What happens as $x$ approaches 0?

75. $f(x) = \dfrac{1 - \cos x}{x}$

76. $f(x) = \dfrac{\sin 4x}{2x}$

## APPLICATIONS

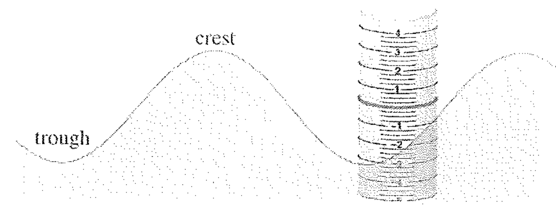
77. **Height of a Wave** As a wave passes by an offshore piling, the height of the water is modeled by the function

$$h(t) = 3 \cos\left(\frac{\pi}{10} t\right)$$

where $h(t)$ is the height in feet above mean sea level at time $t$ seconds.

(a) Find the period of the wave.

(b) Find the wave height, that is, the vertical distance between the trough and the crest of the wave.




78. **Sound Vibrations** A tuning fork is struck, producing a pure tone as its tines vibrate. The vibrations are modeled by the function

$$v(t) = 0.7 \sin(880\pi t)$$

where $v(t)$ is the displacement of the tines in millimeters at time $t$ seconds.

(a) Find the period of the vibration.

(b) Find the frequency of the vibration, that is, the number of times the fork vibrates per second.

(c) Graph the function $v$.

79. **Blood Pressure** Each time your heart beats, your blood pressure first increases and then decreases as the heart rests between beats. The maximum and minimum blood pressures are called the *systolic* and *diastolic* pressures, respectively. Your *blood pressure reading* is written as systolic/diastolic. A reading of 120/80 is considered normal.

A certain person's blood pressure is modeled by the function

$$p(t) = 115 + 25 \sin(160\pi t)$$

where $p(t)$ is the pressure in mmHg (millimeters of mercury), at time $t$ measured in minutes.

(a) Find the period of $p$.

(b) Find the number of heartbeats per minute.

(c) Graph the function $p$.

(d) Find the blood pressure reading. How does this compare to normal blood pressure?

80. **Variable Stars** Variable stars are ones whose brightness varies periodically. One of the most visible is R Leonis; its brightness is modeled by the function

$$b(t) = 7.9 - 2.1 \cos\left(\frac{\pi}{156} t\right)$$

where $t$ is measured in days.

(a) Find the period of R Leonis.

(b) Find the maximum and minimum brightness.

(c) Graph the function $b$.