Name _____ Date _____

Math 111: Final Review 4b

1. You have a group of Z teachers who are anxious to get add days to the school year. The talk about added days can be modeled by the function:

$$S(t) = \frac{Z}{5 + Pe^{-kt}}$$

t = time in hours, $S(t)$ is the number of teachers talking about added days at time t. P and k are constants.

a) Find $t_{25\%}$ (the time at which 25% of the teachers are talking about adding days). Express your answer in terms of P and k.

$$0.25Z = \frac{Z}{5+Pe^{-kt}}$$

$$0.25Z(5+Pe^{-kt}) = Z \quad /0.25Z$$

$$5 + Pe^{-kt} = 4$$

$$Pe^{-kt} = -1$$

$$\ln e^{-kt} = -1/P = -P^{-1}$$

$$\frac{-kt}{-k} = \frac{\ln(-P^{-1})}{-k} = \frac{-\ln(-P)}{-k}$$

$$t = \frac{\ln(-P)}{k}$$

b) Suppose $Z = 40$. Find P if $S(0) = 4$

$$4 = \frac{40}{5+Pe^{-k(0)}}$$

$$20 + 4P = 40 \qquad P = 5$$

$$4P = 20$$

c) Use the value of P that you found in *part b* to find k, if $S = 7$ at $t = 32$ hours. You may leave your answer as an unsimplified logarithm.

$$7 = \frac{40}{5+5e^{-k(32)}}$$

$$35 + 35e^{-32k} = 40$$

$$35e^{-32k} = 5$$

$$\ln e^{-32k} = \ln 0.142857$$

$$-32k = \ln 0.142857$$

$$k = \frac{\ln 0.142857}{-32}$$

2. In a group project in learning theory, a model for proportion P of correct answers after n tests was found to be:

$$P(n) = \frac{R}{1.5 + Te^{kn}}$$ R is a constant.

a) Find $n_{40\%}$ (the number trial at which 40% of the possible responses are correct) in terms of T and n.

$0.4R = \frac{R}{1.5 + Te^{kn}}$

$0.4R(1.5 + Te^{kn}) = \frac{R}{0.4R}$

$1.5 + Te^{kn} = 2.5$

$Te^{kn} = 1$

$e^{kn} = \frac{1}{T} = T^{-1}$

$\ln$

$\frac{kn}{k} = \frac{\ln T^{-1}}{k} = \frac{-\ln T}{k}$

$n = -\frac{\ln T}{k}$

b) Suppose $R = 50$, find T if $P(0) = 5$.

$5 = \frac{50}{1.5 + Te^{k(0)}}$

$5(1.5 + T(1)) = 50$

$7.5 + 5T = 50$

$5T = 42.5$

$T = 8.5$

c) Use the value of T that you found in *part b* to find k, if $S = 26$ at $n = 12$ tests. You may leave your answer as an unsimplified logarithm.

$26 = \frac{50}{1.5 + 8.5e^{k(12)}}$

$26(1.5 + 8.5e^{12k}) = 50$

$1.5 + 8.5e^{12k} = 1.923$

$8.5e^{12k} = 0.423$

$e^{12k} = 0.0498$

$\ln$

$\frac{12k}{12} = \frac{\ln 0.0498}{12}$

$k = \frac{\ln 0.0498}{12}$