Name Mine Date

Math 111: Final Review 4

1. You have a group of Q students who are anxious to get another snow day. The talk about snow can be modeled by the function:

$$S(t) = \frac{Q}{3 + De^{-kt}}$$

t = time in hours, $S(t)$ is the number of students talking about snow at time t. D and k are constants.

a) Find $t_{80\%}$ (the time at which 80% of the students are talking about snow). Express your answer in terms of D and k.

$0.8Q = \frac{Q}{3+De^{-kt_{80\%}}}$

$0.8Q(3+De^{-kt}) = Q$

$3 + De^{-kt} = 1.25$

$De^{-kt} = -1.75$

$\ln e^{-kt} = \ln \frac{-1.75}{D}$

$-kt = \ln\left(\frac{-1.75}{D}\right)$

$t_{80\%} = \frac{\ln\left(\frac{-1.75}{D}\right)}{-k}$

b) Suppose $Q = 800$. Find D if $S(0) = 4$

$4 = \frac{800}{3+De^{-k(0)}}$

$4 = \frac{800}{3+D(1)}$

$4(3+D) = 800$

$3 + D = 200$

$D = 197$

c) Use the value of D that you found in part b to find k, if $S = 10.6$ at $t = 5$. You may leave your answer as an unsimplified logarithm.

$S(5) = 10.6 = \frac{800}{3+197e^{-k(5)}}$

$10.6(3+197e^{-5k}) = 800$

$3 + 197e^{-5k} = 75.5$

$197e^{-5k} = 72.5$

$\ln e^{-5k} = \ln 0.368$

$-5k = \ln 0.368$

$k = \frac{\ln 0.368}{-5} \approx 0.2$

ok

2. In a group project in learning theory, a model for proportion P of correct responses after n trials was found to be:

$$P(n) = \frac{R}{1+e^{-0.2n}}$$ R is a constant.

a) Find $n_{75\%}$ (the number trial at which 75% of the possible responses are correct). ~~Express your answer in terms of R.~~

$0.75R = \frac{R}{1+e^{-0.2n}}$

$\frac{0.75R(1+e^{-.2n})}{0.75R} = \frac{R}{0.75R}$

$1+e^{-0.2n} = \frac{1}{0.75} = 1.\overline{3}$

$-1 \quad -1$

$\ln e^{-0.2n} = \ln 0.\overline{3}$

$\frac{-0.2n}{-.2} = \frac{\ln 0.\overline{3}}{-.2}$

$n = \frac{\ln 0.\overline{3}}{-0.2} = 5.493$

6 trials

b) Find R if $P(0) = 0.415$.

$P(0) = 0.415 = \frac{R}{1+e^{-0.2(0)}}$

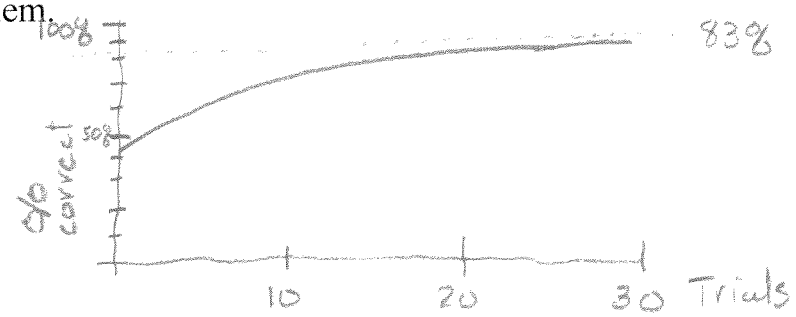
$0.415 = \frac{R}{1+1} = \frac{R}{2}$ $\times 2$

$0.83 = R$

$P(n) = \frac{0.83}{1+e^{-0.2n}}$

c) Use the value of R that you found in part b to graph the on a graphing calculator. Determine the horizontal asymptote of the function. Interpret the meaning of the asymptote in the context of the problem.

No matter how many trials, the subjects could only respond correctly 83% of the time.



d) After how many trials will 60% of the responses be correct?

$0.60 = \frac{0.83}{1+e^{-0.2n}}$

$\frac{0.60(1+e^{-0.2n})}{0.6} = \frac{0.83}{0.6}$

$1+e^{-2n} = \frac{0.83}{0.6} - 1$

$\ln e^{-0.2n} = \ln \frac{0.83}{0.6} - 1$

$\frac{-0.2n}{-0.2} = \frac{\ln\left(\frac{0.83}{0.6} - 1\right)}{-0.2} = 4.794$

~5 trials