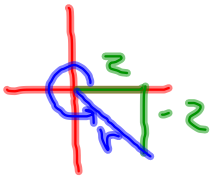
35 standard form Polar Form

$z = a + bi$ $\quad w = r(\cos\theta + i\sin\theta)$

$r = \sqrt{a^2+b^2}$

$\tan\theta = \frac{b}{a}$

A) $z = 2 - 2i$ $\quad \theta \in [0, 2\pi)$



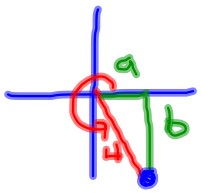
$r = \sqrt{(2)^2+(-2)^2}$

$r = \sqrt{8} = 2\sqrt{2}$

$\theta = \frac{7\pi}{4}$

$w = 2\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

$w = 4\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

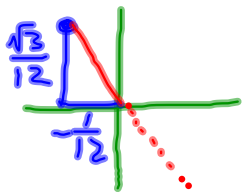


$4\left(+\frac{1}{2} + i\cdot -\frac{\sqrt{3}}{2}\right)$

$2 - 2i\sqrt{3}$

36 c

$z = \frac{1}{3}\left(-\frac{1}{4} + \frac{\sqrt{3}}{4}i\right) = -\frac{1}{12} + \frac{\sqrt{3}}{12}i$



$r = \sqrt{\left(-\frac{1}{12}\right)^2 + \left(\frac{\sqrt{3}}{12}\right)^2} = \sqrt{\frac{1}{144} + \frac{3}{144}} = \sqrt{\frac{4}{144}} = \frac{2}{12} = \frac{1}{6}$

$\tan\theta = \frac{\sqrt{3}/12}{-1/12} = -\sqrt{3} \quad \theta = -\frac{\pi}{3} = \frac{2\pi}{3}$

$w = \frac{1}{6}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

Find $z^6 \rightarrow \quad r^n(\cos n\cdot\theta + i\sin n\theta)$

$\left(\frac{1}{6}\right)^6\left(\cos\left(6\cdot\frac{2\pi}{3}\right) + i\sin\left(6\cdot\frac{2\pi}{3}\right)\right)$

$\frac{1^6}{6^6}(\cos 4\pi + i\sin 4\pi)$

$\frac{1^6}{6^6}(1 + 0i) = \left(\frac{1}{6}\right)^6 + 0i$

part 2: find $\sqrt[4]{z}$ or $z^{1/4}$, $n=4$; $k=0,1,2,3$

$w = \frac{1}{6}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$w_k = r^{1/n}\left(\cos\left(\frac{1}{n}\cdot\theta\right) + i\sin\left(\frac{1}{n}\cdot\theta\right)\right)$

$w_0 = \left(\frac{1}{6}\right)^{1/4}\left(\cos\left(\frac{1}{4}\cdot\frac{2\pi}{3}\right) + i\sin\left(\frac{1}{4}\cdot\frac{2\pi}{3}\right)\right)$

$= \left(\frac{1}{6}\right)^{1/4}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = \left(\frac{1}{6}\right)^{1/4}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$+\frac{2\pi}{n} = \frac{2\pi}{4} = \frac{\pi}{2} = \frac{3\pi}{6}$

$w_1 = \left(\frac{1}{6}\right)^{1/4}\left(\cos\left(\frac{\pi}{6}+\frac{3\pi}{6}\right) + i\sin\left(\frac{\pi}{6}+\frac{3\pi}{6}\right)\right)$

$= \left(\frac{1}{6}\right)^{1/4}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = \left(\frac{1}{6}\right)^{\frac{1}{4}}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$w_2 = \left(\frac{1}{6}\right)^{1/4}\left(\cos\left(\frac{2\pi}{3}+\frac{3\pi}{6}\right) + i\sin\left(\frac{2\pi}{3}+\frac{3\pi}{6}\right)\right)$

$= \left(\frac{1}{6}\right)^{1/4}\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right) = \left(\frac{1}{6}\right)^{1/4}\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$

$w_3 = \left(\frac{1}{6}\right)^{1/4}\left(\cos\left(\frac{7\pi}{6}+\frac{3\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}+\frac{3\pi}{6}\right)\right)$

$= \left(\frac{1}{6}\right)^{1/4}\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) = \left(\frac{1}{6}\right)^{1/4}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$