21 F

$$\tan 15^\circ = \tan(45^\circ - 30^\circ)$$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$

$$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1 \cdot \frac{\sqrt{3}}{3}} = \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}} \cdot \frac{3}{3} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}}$$

$$= \frac{(3-\sqrt{3})}{(3+\sqrt{3})} \cdot \frac{(3-\sqrt{3})}{(3-\sqrt{3})} = \frac{9 - 3\sqrt{3} - 3\sqrt{3} + 3}{9 - 3} = \frac{12 - 6\sqrt{3}}{6}$$

$$= 2 - \sqrt{3} = \frac{6(2-\sqrt{3})}{6}$$

22

$$\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)} = \frac{\cot\alpha + \tan\beta}{\cot\alpha - \tan\beta}$$

$$\frac{(\cos\alpha\cos\beta + \sin\alpha\sin\beta)}{(\cos\alpha\cos\beta - \sin\alpha\sin\beta)} \Big/ \frac{(\sin\alpha\cos\beta)}{(\sin\alpha\cos\beta)}$$

$$\frac{\frac{\cos\alpha\cos\beta}{\sin\alpha\cos\beta} + \frac{\sin\alpha\sin\beta}{\sin\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\sin\alpha\cos\beta} - \frac{\sin\alpha\sin\beta}{\sin\alpha\cos\beta}} = \frac{\frac{\cos\alpha}{\sin\alpha} + \frac{\sin\beta}{\cos\beta}}{\frac{\cos\alpha}{\sin\alpha} - \frac{\sin\beta}{\cos\beta}}$$

$$\downarrow$$

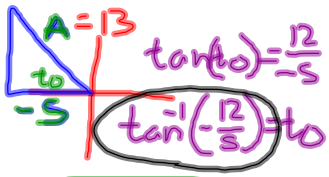
$$\frac{\cot\alpha + \tan\beta}{\cot\alpha - \tan\beta}$$

$$A\cos(t-t_0) = -5\cos t + 12\sin t$$

$$\cos(t-t_0) = \frac{-5}{A}\cos t + \frac{12}{A}\sin t$$

$$\cos(t-t_0) = \cos t\cos t_0 + \sin t\sin t_0 = \frac{-5}{A}\cos t + \frac{12}{A}\sin t$$

So $\cos t_0 = \frac{-5}{A}$ $\sin t_0 = \frac{12}{A}$ 12



$\tan(t_0) = \frac{12}{-5}$

$\tan^{-1}\left(\frac{12}{-5}\right) = t_0$

$13\cos\left(t-\tan^{-1}\left(\frac{12}{-5}\right)\right)$

$13\cos\left(t+\tan^{-1}\left(\frac{12}{5}\right)\right)$

$A = \sqrt{(-5)^2+12^2}$

$A = \sqrt{169} = 13$

$\tan^{-1}\left(\frac{12}{-5}\right) = -\tan^{-1}\left(\frac{12}{5}\right)$