Math 112: #1 A/B

1. Suppose two moon cities have the same longitude and their latitudes are 37° and 54°. Assuming the radius of the moon is 1,737 *km*. Find the distance between the two cities as measured along the surface of the moon. State the exact answer, then give a decimal answer rounded to ~~four~~ two decimal places.

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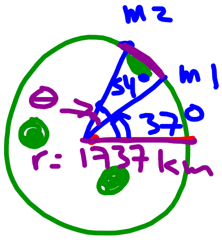


$\theta = 54^\circ - 37^\circ = 17^\circ$

$S = r\theta = 1737 \cdot \frac{17\pi}{180} = 164.05\pi \text{ km}$

radians

$= 515.38 \text{ km}$

$\theta = 17^\circ \times \frac{\pi}{180^\circ} = \frac{17\pi}{180}$

2. Suppose two cities have the same longitude and their latitudes are 45° 48' (Scappoose) and 65° 12' (Great Bear Lake). Assuming the radius of the Earth is 3,955 *miles*, find the distance between the two cities as measured along the surface of the Earth. State the exact answer, then give a decimal answer rounded to two decimal places.

$65^\circ 12'$ GBL 65.2° ($\frac{12}{60}$)

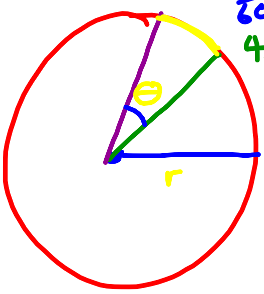
$45^\circ 48'$ Scap $= 45.8^\circ$ ($\frac{48}{60}$)

$37^\circ\ \frac{15'}{60}\ \cancel{\frac{46''}{3600}}$

37.25

$.0127$

37.2627°



$\theta = 65.2^\circ - 45.8^\circ$

$= 19.4^\circ$

$= 19.4^\circ \cdot \frac{\pi}{180^\circ}$

$= 0.10\overline{7}\pi$ radians

$S = r\theta = 3955 \times 0.10\overline{7}\pi$

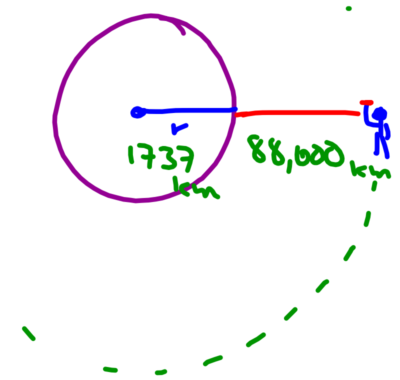
$= 426.26\overline{1}\pi$ miles

$= 1339.14$ miles

Math 112: #2 A/B

1. Suppose Dr. Evil's spaceship is in lunarsynchronous orbit around the Moon. It remains above the same point on the moon's equator about 88,000 *km* above the surface. Assuming the radius of the moon is 1,737 *km*. Find the following. State the exact answer, then give a decimal answer rounded to ~~two~~ 4 decimal places.

a. The angular speed of the spaceship in radians per hour. (A moon day, the length of time it takes the moon to spin around its axis is about 27 days - a sidereal day).



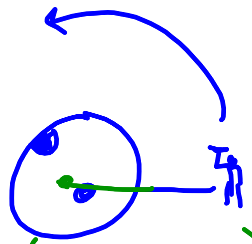
angular speed $= \omega = \dfrac{\theta}{t}$

$$= \frac{\cancel{2}\pi}{27\cdot\cancel{24}\,12}$$

$$= \frac{\pi}{27\cdot 12} = \frac{\pi}{324} \text{ radians/hour}$$

$$= 0.0097 \text{ radians/hour}$$

b. The linear speed of the spaceship in kilometers per hour.



$r = (1737 + 88{,}000)$ km

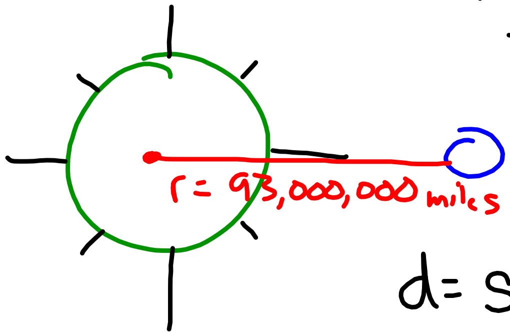
linear Speed $= \dfrac{S}{t} = V$

$$S = r\theta = 89{,}737 \cdot 2\pi = 179{,}474\pi \text{ km}$$

$$V = \frac{179{,}474\pi}{27\cdot 24} = 276.97\pi \text{ km/hour}$$

$$= 870.11 \text{ km/hour}$$

2. Find the distance that the earth travels in one day in its path around the sun. Assume that a year has 365 days and that the path of the earth around the sun is a circle of radius of 93 million miles. State the exact answer, then give a decimal answer rounded to two decimal places. + find the linear speed of the earth in miles/day.



$d = s = r\theta = 93{,}000{,}000 \cdot 2\pi$

$= 186{,}000{,}000\pi$ miles

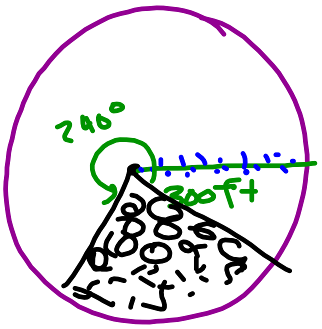
Linear Speed $= \frac{s}{t} = \frac{186{,}000{,}000\pi \text{ miles}}{365 \text{ days}} = 509{,}589.04\pi$ miles/day

$= 1{,}600{,}921.19$ mile/day

Math 112: #3 A/B

1. An irrigation system uses a straight sprinkler pipe 300*ft* long that pivots around a central point. Due to an obstacle the pipe only pivots 240°.

a. Find the area irrigated by this system. Sector Area



$SA = \frac{1}{2}\theta r^2$

$SA = \frac{1}{2} \cdot \frac{4\pi}{3} \cdot (300ft)^2$

$= \frac{2\pi}{3} \cdot 90000ft^2$

$= 60{,}000\pi \; ft^2$

$= 188{,}495.56 \; ft^2$

$240^\circ \cdot \frac{\pi}{180^\circ} = \frac{4\pi}{3}$

$4(60^\circ)$

$4\left(\frac{\pi}{3}\right)$

b. How fast will the sprinkler need to turn to irrigate 100,000 square feet in an hour?

Sector Area

Find θ

$\frac{\theta}{1hr}$ is angular Velocity

$SA = \frac{1}{2}\theta r^2$

$$\frac{100{,}000}{\frac{1}{2}\cdot 300^2} = \frac{\cancel{1}}{\cancel{2}}\theta(\cancel{300ft})^2$$

$2.2\bar{2}$ radians $= \theta$

$2.2\bar{2}$ radians/hour

c. How fast will the sprinkler need to turn to irrigate one square mile in an hour?

Sector Area

$(5280ft)^2$

$$\frac{27{,}878{,}400\cancel{ft^2}}{\frac{1}{2}\cdot(300\cancel{ft})^2} = \frac{1}{2}\theta(300ft)^2$$

619.52 radians $= \theta$

$$\omega = \frac{\theta}{t} = \frac{619.52 \text{ radians}}{1 \text{ hour}} = 619.52 \text{ radians/hour}$$

2. A windshield wiper fixed to a window is $40cm$ long. Assume there is no gap between the pivot point and the wiper. What angular speed is necessary for the wiper to clear $300cm^2$ per second?