Section 9.3 Law of Cosines

Law of Sines
- ASA, SAA
- SSA - ambiguous

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines
- SSS, SAS

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$+2bc\cos A \qquad +2bc\cos A$$

$$a^2 + 2bc\cos A = b^2 + c^2 - a^2$$

$$-a^2 \qquad\qquad 2bc \qquad\qquad 2bc$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

what happens if $C = 90^\circ$ ↑ right triangle

$$\cos 90^\circ = ?$$

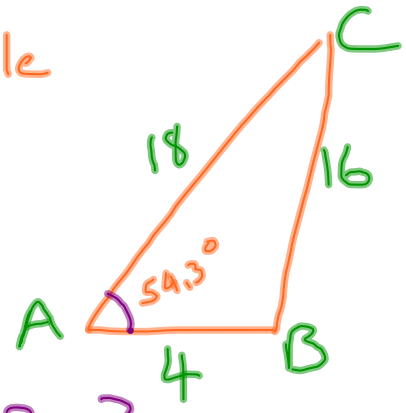
$$= 1$$

$$= 0$$

$$c^2 = a^2 + b^2 - 2ab(0)$$

$$c^2 = a^2 + b^2$$ ← pythagorean Thm is a special case of the Law of Cosines.

SSS Example

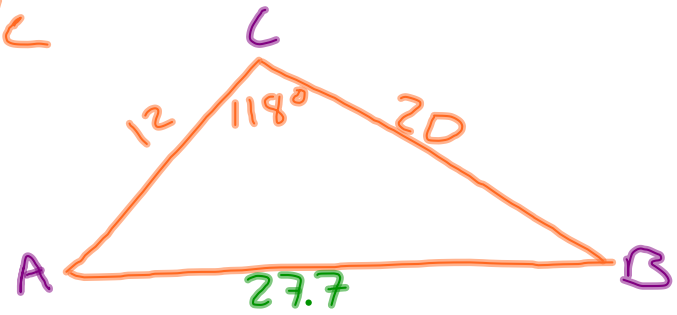


$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{18^2+4^2-16^2}{2\cdot 18\cdot 4} = 0.58\overline{3}$$

$$\cos A = 0.58\overline{3}$$

$$\cos^{-1}(0.58\overline{3}) = A$$

SAS example



$$c^2 = a^2+b^2-2ab\cos C$$

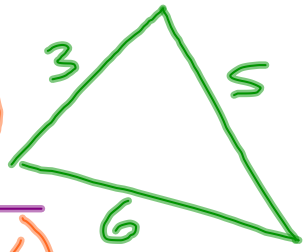
$$c^2 = 20^2+12^2-2\cdot 20\cdot 12\cdot \cos 118^\circ$$

$$\sqrt{c^2} = \sqrt{769.3}$$

$$c = 27.7$$

Area example

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$



$$s = \frac{a+b+c}{2}$$

$$\text{Area} = \sqrt{7(7-3)(7-5)(7-6)}$$

$$s = \frac{3+5+6}{2}$$

$$\text{Area} = \sqrt{7 \cdot 4 \cdot 2 \cdot 1} = \sqrt{56} = 7.5u^2$$

$$s = \frac{14}{2} = 7$$