

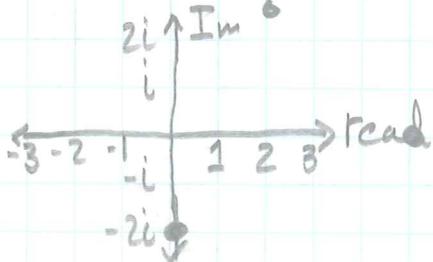
Notes 8.3A: Polar Form of Complex Numbers

- A Complex Number has form: $a+bi$

$$i = \sqrt{-1} \quad i^2 = -1$$

real \uparrow imaginary

- Complex Numbers are plotted on the complex plane

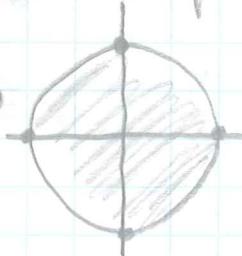


$a+bi$ is plotted as the point (a, b) on the complex plane

$2+3i$	$(2, 3)$	$-2i$	$(0, -2)$
$-1+4i$	$(-1, 4)$	5	$(5, 0)$

Graphing sets of complex numbers

$$\{z \mid |z| \leq 2\}$$



$$\{z = a+bi \mid a \geq 0, b \leq 0\}$$

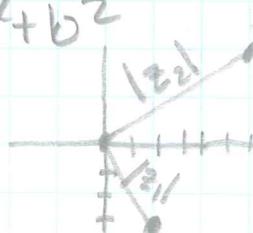


$\rightarrow 3^{\text{rd}}$

Modulus or Absolute Value of a Complex Number

$$|z| = \sqrt{a^2 + b^2} \quad z_1 = 2 - 3i \quad |z| = \sqrt{2^2 + (-3)^2} = \sqrt{13}$$

$\rightarrow 2^{\text{nd}}$



$$z_2 = 4\sqrt{3} + 4i \quad |z| = \sqrt{(4\sqrt{3})^2 + 4^2}$$

$$= \sqrt{48 + 16} = \sqrt{64} = 8$$

Polar Form of Complex Numbers

$$z = r(\cos \theta + i \sin \theta) \quad \text{where } r = |z| = \sqrt{a^2 + b^2}$$

$$\text{and } \tan \theta = \frac{b}{a}$$

θ is not unique, but must be in the right Quadrant

Write the complex numbers in polar form:

$$\bullet 1 + \sqrt{3}i \quad |z| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

~~+1~~

$$\tan \theta = \frac{\sqrt{3}}{1} \Rightarrow \tan^{-1} \sqrt{3} = \theta = \frac{\pi}{3}$$

$$z = 2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$\bullet 1 - i$$

~~+1~~

$$|z| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$\tan \theta = \frac{-1}{1} \rightarrow \tan^{-1}(-1) = -\frac{\pi}{4} \text{ or } \frac{7\pi}{4}$$

$$z = \sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$$

$$\bullet -2\sqrt{3} + 2i$$

~~+2~~

$$|z| = \sqrt{(-2)^2 + (2\sqrt{3})^2} = \sqrt{4+12} = 4$$

$$\tan^{-1} \left(\frac{2}{-2\sqrt{3}} \right) = \tan^{-1} \left(-\frac{1}{\sqrt{3}} \right) = -\frac{\pi}{6} \rightarrow \frac{5\pi}{6}$$

$$z = 4 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$$

$$\bullet 5i = 0 + 5i$$

~~+0~~

$$|z| = \sqrt{(0)^2 + (5)^2} = 5$$

$$\tan^{-1} \left(\frac{5}{0} \right) = \tan^{-1}(\text{und}) = \frac{\pi}{2}$$

$$z = 5 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$

$$\bullet 3 + 4i$$

$$|z| = \sqrt{3^2 + 4^2} = 5$$

$$\tan \theta = \frac{4}{3} \quad \tan^{-1} \left(\frac{4}{3} \right) = \theta$$

$$z = 5 \left(\cos \left(\tan^{-1} \left(\frac{4}{3} \right) \right) + i \sin \left(\tan^{-1} \left(\frac{4}{3} \right) \right) \right)$$

#43

Notes Section 8.3B: DeMoivre's Thm

$$z_1 = r_1(\cos \theta_1 + i \sin \theta_1) \quad z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$$

$$z_1 z_2 = r_1 r_2 (\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2))$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} (\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2))$$

#55) $z_1 = 3(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}) \quad z_2 = 5(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3})$

$$z_1 z_2 = 3 \cdot 5 \left(\cos \frac{\pi}{6} + \frac{4\pi}{3} + i \sin \frac{\pi}{6} + \frac{4\pi}{3} \right) \quad \frac{\pi}{6} + \frac{4\pi}{3} = \frac{9\pi}{6} = \frac{3\pi}{2}$$

$$15 \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right)$$

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{3}{5} \left(\cos \frac{\pi}{6} - \frac{4\pi}{3} + i \sin \frac{\pi}{6} - \frac{4\pi}{3} \right) \quad \frac{\pi}{6} - \frac{4\pi}{3} = -\frac{7\pi}{6} = \\ &\quad 3/5 \left(\cos -\frac{7\pi}{6} + i \sin -\frac{7\pi}{6} \right) = 3/5 \left(\cos \frac{7\pi}{6} - i \sin \frac{7\pi}{6} \right) \end{aligned}$$

De Moivre's Thm $z^n = r^n (\cos n\theta + i \sin n\theta)$

#70) $(1 - \sqrt{3}i)^5 \quad z = 2 \left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right) \quad z^5 = 2^5 \left(\cos 5 \left(\frac{5\pi}{3} \right) + i \sin 5 \left(\frac{5\pi}{3} \right) \right)$

$\cancel{+}$ $r = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{4} = 2$ $32 \left(\cos \frac{25\pi}{3} + i \sin \frac{25\pi}{3} \right) = 16 + 16\sqrt{3}i$
 $\tan \theta = -\sqrt{3}/1 \Rightarrow \theta = -\frac{\pi}{3} = \frac{5\pi}{3}$ $32 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) = 32 \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right)$

n^{th} Roots of Complex Numbers $w_k = r^{\frac{1}{n}} \left[\cos \frac{\theta + 2k\pi}{n} + i \sin \frac{\theta + 2k\pi}{n} \right]$

• 1 $r = r^{\frac{1}{n}}$ for each root $k = 0, 1, \dots, n-1$

• 2 The 1st root ($k=0$) has argument $\frac{\theta}{n}$

• 3 Add $\frac{2\pi}{n}$ each time to get next argument

$$\#82) (4\sqrt{3} + 4i)$$

cube roots
don't forget
to graph

Solving Equations using n^{th} roots formula

$$\#92) z^8 - i = 0 \quad +$$

8th
roots
of z

$$z^8 = i \quad (0, 1)$$
$$r = 1$$
$$\Theta = \frac{\pi}{2}$$