Notes 7.3: Double-Angle, Half-Angle & Product-Sum Formulas

Double Angle Formulas:

- $\sin 2x = 2\sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$
- $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

Formulas for Lowering Powers

- $\sin^2 x = \dfrac{1 - \cos 2x}{2}$
- $\cos^2 x = \dfrac{1 + \cos 2x}{2}$
- $\tan^2 x = \dfrac{1 - \cos 2x}{1 + \cos 2x}$

Half-Angle Formulas

- $\sin \dfrac{u}{2} = \pm\sqrt{\dfrac{1 - \cos u}{2}}$
- $\cos \dfrac{u}{2} = \pm\sqrt{\dfrac{1 + \cos u}{2}}$
- $\tan \dfrac{u}{2} = \dfrac{1 - \cos u}{\sin u} = \dfrac{\sin u}{1 + \cos u}$

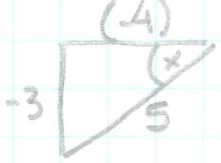
Product to Sum Formulas

- $\sin u \cdot \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$
- $\cos u \cdot \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$
- $\cos u \cdot \cos v = \frac{1}{2}[\cos(u+v) + \cos(u-v)]$
- $\sin u \cdot \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$

Sum to Product Formulas

- $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$
- $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$
- $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$
- $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

ex: 7) $\sin x = -\frac{3}{5}$ in III find $\sin 2x, \cos 2x, \tan 2x$



$\cos x = -4/5$

$\tan x = 3/4$

$\sin 2x = 2\sin x\cos x = 2\cdot -3/5 \cdot -4/5$

$= 24/25$

$\cos 2x = \cos^2 x - \sin^2 x$

$(-4/5)^2 - (-3/5)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\cdot 3/4}{1-9/16} = \frac{6/4}{7/16}$

$\frac{24}{7}$