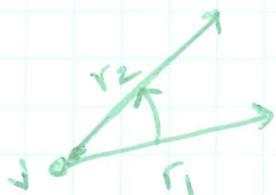


Section 6.1: Angle Measure

- Angle Measure - degrees & radians
- Angles in Standard Position \perp
- Arc Length (s)
- Area of circular sectors
- Circular Motion

Angle Measure



Angle = amount of rotation from $r_1 \rightarrow r_2$
measure in degrees: $360^\circ/\text{circle}$
or radians $2\pi/\text{circle}$

1 radian = 57.295° = the ~~dis~~ length of arc
the angle that corresponds
central to an arc length
= to the radius

see
wikipedia
page

$C = 2\pi r$
so there are 2π radians
in a circle.

$$\begin{aligned} 2\pi &= 360^\circ \\ \pi &= 180^\circ \\ \pi/2 &= 90^\circ \\ \pi/3 &= 60^\circ \\ \pi/4 &= 45^\circ \\ \pi/6 &= 30^\circ \end{aligned}$$

• converting degrees \rightarrow radians $\times \frac{\pi}{180}$

• converting radians \rightarrow degrees $\times \frac{180}{\pi}$

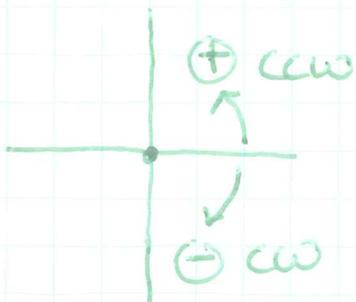
ex: $210^\circ \cdot \frac{\pi}{180} = \frac{7\pi}{6}$ rad or $\frac{7\pi}{6}$

• $407^\circ \cdot \frac{\pi}{180} \approx 2.261\pi$ or ≈ 7.1

• $\frac{5\pi}{3} \cdot \frac{180}{\pi} = \frac{5 \cdot 180}{3} = 300^\circ$

• $23.2\pi \cdot \frac{180}{\pi} = 4176^\circ$

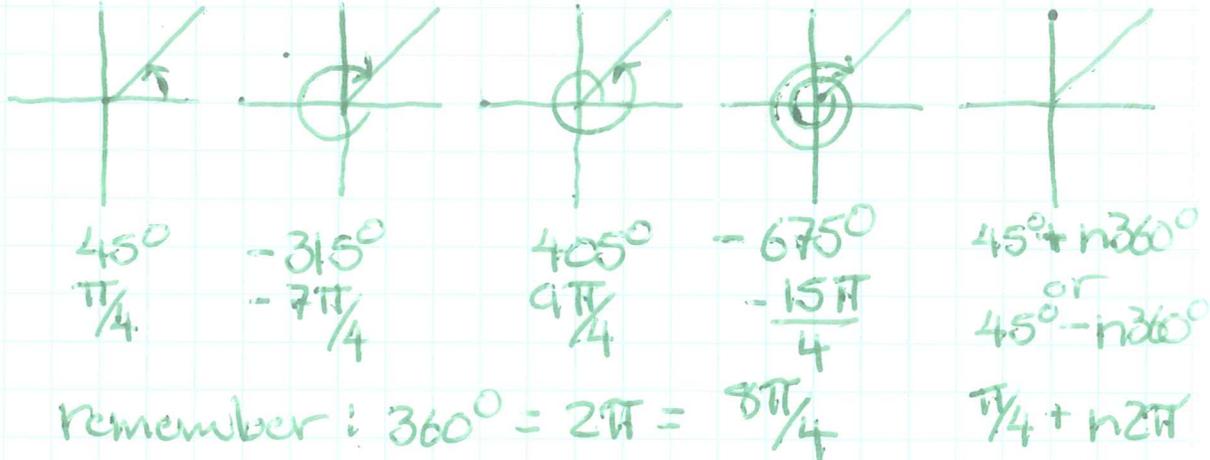
• $4.27 \cdot \frac{180}{\pi} \approx 244.65^\circ$



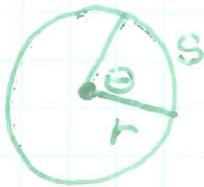
Angles in Standard Position

vertex at $(0,0)$ initial ray at x-axis
($0^\circ, 0$ rad)

coterminal angles have same terminal rays



Arc Length (s)



Arc length (s) that subtends a central angle of θ radians ← is always

$$s = r\theta$$

$$\theta = \frac{s}{r} \quad \text{if } r = \frac{s}{\theta}$$

also used

ex:

- find s if $r = 5_m$ and $\theta = \frac{\pi}{3}$

$$s = 5_m \cdot \frac{\pi}{3} = \frac{5\pi}{3} m \approx 5.2_m$$
- find θ if $r = 4_{in}$ and the arc length = 7_{in}

$$\theta = \frac{7_{in}}{4_{in}} = \frac{7}{4} = 1.75 \text{ radians}$$
- find r if an arc of length 9_{ft} subtends a central angle of 240°

$$\theta = \frac{240^\circ}{360^\circ} \cdot \frac{\pi}{1} = \frac{2\pi}{3}$$

$$r = \frac{9_{ft}}{\frac{2\pi}{3}} = \frac{27_{ft}}{2\pi} \approx 2.15_{ft}$$



Area of a Sector

$$A_{\text{sector}} = \frac{\theta}{2\pi} \cdot \pi r^2 = \frac{1}{2} r^2 \theta$$

\leftarrow radians
 \uparrow
 % circle

$$A = 2\pi r$$

circle

ex: find the area of a sector if $r = 6\text{cm}$ and $\theta = +\frac{5\pi}{3}$

$$A = \frac{1}{2} (6\text{cm})^2 \cdot \frac{5\pi}{3}$$

$$A = \frac{1}{2} \cdot \overset{+36}{36}\text{cm}^2 \cdot \frac{5\pi}{3} = 30\pi\text{cm}^2$$

$$\approx 94.25\text{cm}^2$$



Linear & Angular Speed

if $s = r\theta$ is the distance a point travels in time t ,

then: Linear Speed $v = \frac{s}{t}$

Angular Speed $\omega = \frac{\theta}{t}$

($r = 15\text{in}$)

ex: if a wheel comes off my car rotating at 120 rpm, how fast was I going in mph?

$$s? = r\theta = 15\text{in} (2\pi \cdot 120) = 3600\pi = 11309.7\text{in}$$

$$v = 11,309.7\text{in}/\text{min} \times 60\text{min}/1\text{hr} \times \frac{1\text{ft}}{12\text{in}} \times \frac{1\text{mi}}{5280\text{ft}}$$

$$10.7\text{mi}/\text{hr}$$