LESSON 1-3

Properties of Numbers

Then

- You used the order of operations to simplify expressions.

Now

1. Recognize the properties of equality and identity. ✓
2. Recognize the Commutative and Associative Properties. ✓

Why?

- Natalie lives 32 miles away from the mall. The distance from her house to the mall is the same as the distance from the mall to her house. This is an example of the Reflexive Property.

1 Properties of Equality and Identity

The expressions $4k + 8k$ and $12k$ are called **equivalent expressions** because they represent the same number. The properties below allow you to write an equivalent expression for a given expression.

KeyConcept Properties of Equality

Property	Words	Symbols	Examples
Reflexive Property	Any quantity is equal to itself.	For any number a, $a = a$.	$5 = 5$ $4 + 7 = 4 + 7$
Symmetric Property	If one quantity equals a second quantity, then the second quantity equals the first.	For any numbers a and b, if $a = b$, then $b = a$.	If $8 = 2 + 6$, then $2 + 6 = 8$.
Transitive Property	If one quantity equals a second quantity and the second quantity equals a third quantity, then the first quantity equals the third quantity.	For any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.	If $6 + 9 = 3 + 12$ and $3 + 12 = 15$, then $6 + 9 = 15$.
Substitution Property	A quantity may be substituted for its equal in any expression.	If $a = b$, then a may be replaced by b in any expression.	If $n = 11$, then $4n = 4 \cdot 11$

$y = 3x + 2$

$3x + 2 = y$

The sum of any number and 0 is equal to the number. Thus, 0 is called the **additive identity**.

KeyConcept Addition Properties

Property	Words	Symbols	Examples
Additive Identity	For any number a, the sum of a and 0 is a.	$a + 0 = 0 + a = a$	$2 + 0 = 2$ $0 + 2 = 2$
Additive Inverse	A number and its opposite are additive inverses of each other.	$a + (-a) = 0$	$3 + (-3) = 0$ $4 - 4 = 0$

Self-Check Practice

There are also special properties associated with multiplication. Consider the following equations.

$4 \cdot n = 4$

The solution of the equation is 1. Since the product of any number and 1 is equal to the number, 1 is called the **multiplicative identity**.

$6 \cdot m = 0$

The solution of the equation is 0. The product of any number and 0 is equal to 0. This is called the **Multiplicative Property of Zero**.

Two numbers whose product is 1 are called **multiplicative inverses** or **reciprocals**. Zero has no reciprocal because any number times 0 is 0.

KeyConcept Multiplication Properties

Property	Words	Symbols	Examples
Multiplicative Identity	For any number a, the product of a and 1 is a.	$a \cdot 1 = a$ $1 \cdot a = a$	$14 \cdot 1 = 14$ $1 \cdot 14 = 14$
Multiplicative Property of Zero	For any number a, the product of a and 0 is 0.	$a \cdot 0 = 0$ $0 \cdot a = 0$	$9 \cdot 0 = 0$ $0 \cdot 9 = 0$
Multiplicative Inverse	For every number $\frac{a}{b}$, where $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that the product of $\frac{a}{b}$ and $\frac{b}{a}$ is 1.	$\frac{a}{b} \cdot \frac{b}{a} = 1$ $\frac{b}{a} \cdot \frac{a}{b} = 1$	$\frac{4}{5} \cdot \frac{5}{4} = \frac{20}{20}$ or 1 $\frac{5}{4} \cdot \frac{4}{5} = \frac{20}{20}$ or 1

$\frac{3}{4} \cdot \frac{4}{3} = 1$

$\frac{12}{12}$

Example 1 Evaluate Using Properties

Evaluate $7(4-3)-1+5\cdot\frac{1}{5}$. Name the property used in each step.

$7(4-3)-1+5\cdot\frac{1}{5}=7(1)-1+5\cdot\frac{1}{5}$	Substitution: $4-3=1$
$=7-1+5\cdot\frac{1}{5}$	Multiplicative Identity: $7\cdot 1=7$
$=7-1+1$	Multiplicative Inverse: $5\cdot\frac{1}{5}=1$
$=6+1$	Substitution: $7-1=6$
$=7$	Substitution: $6+1=7$

GuidedPractice

Name the property used in each step.

1A. $2\cdot 3+(4\cdot 2-8)$

$=2\cdot 3+(8-8)$?

$=2\cdot 3+(0)$?

$=6+0$?

$=6$?

1B. $7\cdot\frac{1}{7}+6(15\div 3-5)$

$=7\cdot\frac{1}{7}+6(5-5)$?

$=7\cdot\frac{1}{7}+6(0)$?

$=1+6(0)$?

$=1+0$?

$=1$?

2 Use Commutative and Associative Properties Nikki walks 2 blocks to her friend Sierra's house. They walk another 4 blocks to school. At the end of the day, Nikki and Sierra walk back to Sierra's house, and then Nikki walks home.

The distance from Nikki's house to school	equals	the distance from the school to Nikki's house.
$2+4$	$=$	$4+2$

This is an example of the **Commutative Property** for addition.

KeyConcept Commutative Property

Words	The order in which you add or multiply numbers does not change their sum or product.
Symbols	For any numbers a and b, $a+b=b+a$ and $a\cdot b=b\cdot a$.
Examples	$4+8=8+4$ $7\cdot 11=11\cdot 7$

An easy way to find the sum or product of numbers is to group, or associate, the numbers using the **Associative Property**.

KeyConcept Associative Property

Words	The way you group three or more numbers when adding or multiplying does not change their sum or product.
Symbols	For any numbers a, b, and c, $(a+b)+c=a+(b+c)$ and $(ab)c=a(bc)$.
Examples	$(3+5)+7=3+(5+7)$ $(2\cdot 6)\cdot 9=2\cdot(6\cdot 9)$

Example 3 Use Multiplication Properties

Evaluate $5 \cdot 7 \cdot 4 \cdot 2$ using the properties of numbers. Name the property used in each step.

$5 \cdot 7 \cdot 4 \cdot 2 = 5 \cdot 2 \cdot 7 \cdot 4$ Commutative (×)

$= (5 \cdot 2) \cdot (7 \cdot 4)$ Associative (×)

$= 10 \cdot 28$ Substitution

$= 280$ Substitution

Guided Practice

Evaluate each expression using the properties of numbers. Name the property used in each step.

3A. $2.9 \cdot 4 \cdot 10$

3B. $\frac{5}{3} \cdot 25 \cdot 3 \cdot 2$