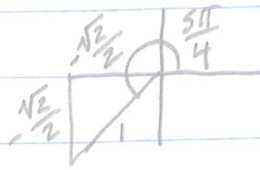


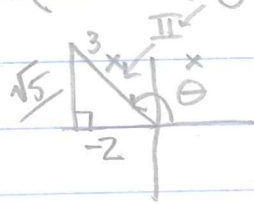
Final Review

1a) $\csc \frac{5\pi}{4} = -\sqrt{2}$



$$\csc = \frac{1}{\sin} = \frac{1}{-\frac{\sqrt{2}}{2}} = -\sqrt{2}$$

b) $\sin(\cos^{-1}(-\frac{2}{3})) = \frac{\sqrt{5}}{3}$



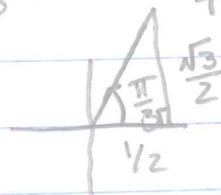
$$(-2)^2 + b^2 = 3^2$$

$$4 + b^2 = 9 \quad b^2 = 5 \quad b = \sqrt{5}$$

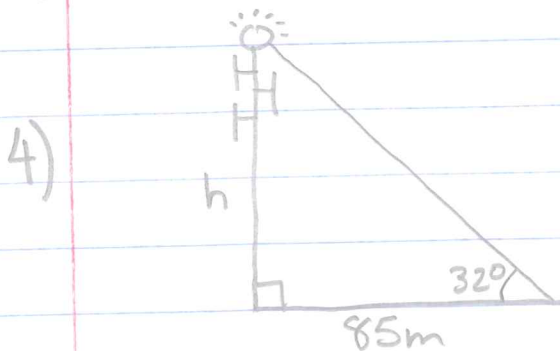
$$\cos^{-1}(-\frac{2}{3}) = \theta \quad \sin \theta = \frac{\sqrt{5}}{3}$$

2) $540^\circ \cdot \frac{\pi}{180^\circ} = \frac{540}{180} \pi = 3\pi$

$$-225^\circ \cdot \frac{\pi}{180^\circ} = -\frac{225}{180} \pi = -\frac{5\pi}{4}$$



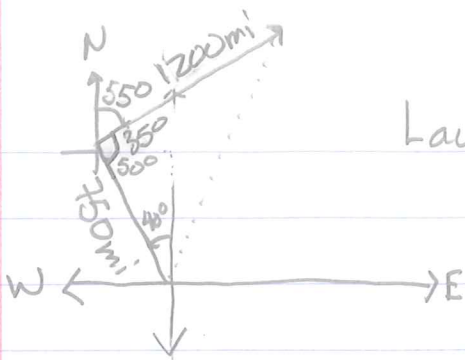
3) $\cos(\frac{\pi}{3}) = \frac{1}{2}$ $\cos = x = \frac{1}{2}$



$$\tan 32^\circ = \frac{h}{85}$$

$$85 \tan 32^\circ = h = 53.1 \text{ m}$$

5)



Law of Cos: $c^2 = a^2 + b^2 - 2ab \cos C$

$$c^2 = 750^2 + 1200^2 - 2 \cdot 750 \cdot 1200 \cos(85^\circ)$$

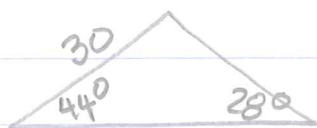
$$c^2 = 1,845,619.7$$

$$c = 1358.5 \text{ miles}$$

$$2.5 \times 300 = 750$$

$$4 \cdot 300 = 1200$$

6a)



$$C = 180^\circ - 28^\circ - 44^\circ = 108^\circ$$

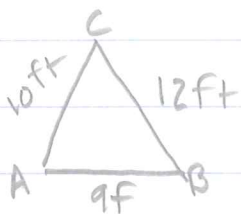
Law of Sin: $\frac{\sin 28^\circ}{30} = \frac{\sin 44^\circ}{a}$

$$\frac{\sin 28^\circ}{30} = \frac{\sin 108^\circ}{c}$$

$$a = \frac{30 \sin 44^\circ}{\sin 28^\circ} = 44.4$$

$$c = \frac{30 \sin 108^\circ}{\sin 28^\circ} = 60.8$$

b)



Law of Cos:

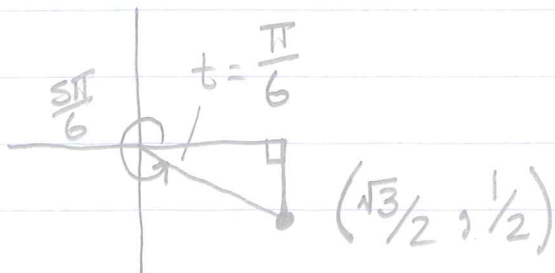
$$\cos A = \frac{10^2 + 9^2 - 12^2}{2 \cdot 10 \cdot 9} = 0.205$$

$$C = 180^\circ - 78.1^\circ - 54.6^\circ \quad \cos^{-1}(0.205) = 78.1^\circ$$

$$C = 47.3^\circ$$

$$\cos B = \frac{9^2 + 12^2 - 10^2}{2 \cdot 9 \cdot 12} \quad \cos^{-1}(\) = 54.6^\circ$$

7)



$$8) a) \cos(\cos^{-1}(x)) = x$$

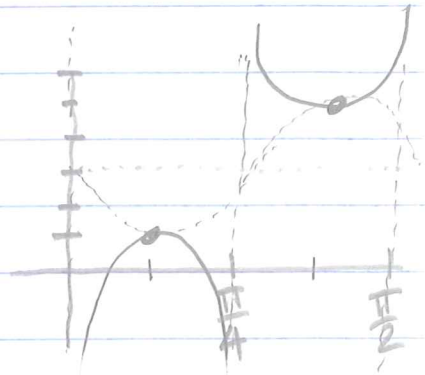
$$-\infty < x < \infty$$

$$b) \sin^{-1}(\sin(x)) = x$$

$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

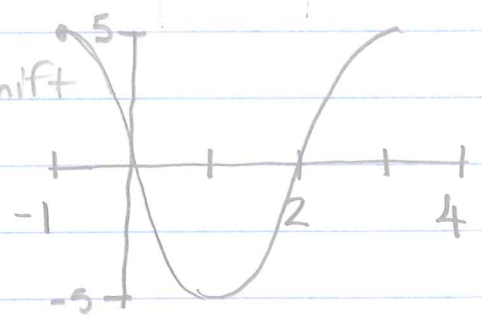
$$9) y = 3 - 2 \csc(4x)$$

a) $\uparrow 3$ \uparrow \uparrow \uparrow \uparrow \uparrow
 flip amp period = $\frac{2\pi}{4} = \frac{\pi}{2}$



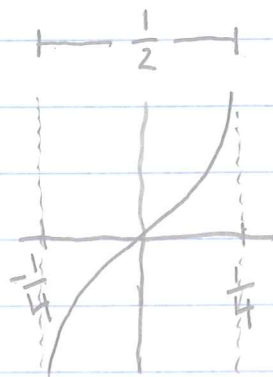
$$b) y = 5 \cos\left(\frac{\pi}{2}x + 1\right)$$

\uparrow amp \uparrow Per = $\frac{2\pi}{\pi/2} = 4$ Phase shift left 1



$$c) y = \tan(2\pi x)$$

$$\text{Per} = \frac{\pi}{2\pi} = \frac{1}{2}$$



$$10) a) \tan\left(\frac{\pi}{2} - x\right) = \cot(x)$$

$$\frac{\sin \frac{\pi}{2} - x}{\cos \frac{\pi}{2} - x} = \frac{\overset{=0}{\sin \frac{\pi}{2}} \cos x - \overset{=0}{\cos \frac{\pi}{2}} \sin x}{\overset{=1}{\cos \frac{\pi}{2}} \cos x + \overset{=0}{\sin \frac{\pi}{2}} \sin x} = \frac{\cos x}{\sin x} = \cot x$$

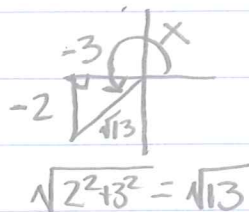
$$b) \sin\left(\frac{\pi}{2} - x\right) = \overset{=1}{\sin \frac{\pi}{2}} \cos x - \overset{=0}{\cos \frac{\pi}{2}} \sin x = \cos x$$

$$11a) \sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$= -\frac{2}{\sqrt{13}} \cdot -\frac{3}{\sqrt{13}} + \frac{3}{5} \cdot -\frac{4}{5}$$

$$= \frac{6}{13} - \frac{12}{25}$$

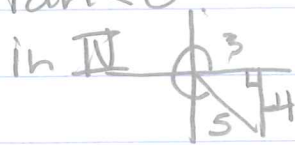
in III $\tan x = \frac{2}{3}$ $\cos < 0$ $\cos y = \frac{3}{5}$
 $\cos < 0$ $\tan < 0$



$$\sqrt{2^2+3^2} = \sqrt{13}$$

$$\sin x = \frac{-2}{\sqrt{13}}$$

$$\cos x = \frac{-3}{\sqrt{13}}$$



$$3^2+b^2=5^2$$

$$b^2=25-9=16$$

$$b=4 \quad \sin y = \frac{-4}{5}$$

$$b) \cos \frac{\pi}{12} = \cos \left(\frac{4\pi}{12} - \frac{3\pi}{12} \right) = \cos \left(\frac{\pi}{3} - \frac{\pi}{4} \right)$$

$$= \cos \frac{\pi}{3} \cos \frac{\pi}{4} + \sin \frac{\pi}{3} \sin \frac{\pi}{4}$$

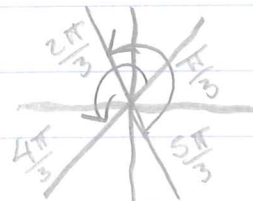
$$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$$

$$12a) \tan^2 x - 3 = 0$$

$$\tan^2 x = 3$$

$$\tan x = \pm \sqrt{3}$$

$$x = \frac{\pi}{3} + k\pi \quad \text{or} \quad -\frac{\pi}{3} + k\pi$$



$$\ast b) 2\cos(3\theta) - 1 = 0$$

$$\cos 3\theta = \frac{1}{2}$$

$$\frac{1}{3} 3\theta = \frac{\pi}{3} + 2k\pi$$

$$\frac{1}{3} 3\theta = \frac{2\pi}{3} + 2k\pi$$

$$\theta = \frac{\pi}{9} + \frac{2k\pi}{3} \quad \text{and} \quad \frac{2\pi}{9} + \frac{2k\pi}{3}$$

$$u = \langle 2, 4 \rangle \quad v = \langle -1, 1 \rangle$$

13 a) $3u - 4v = \langle 6, 12 \rangle - \langle -4, 4 \rangle = \langle 10, 8 \rangle$

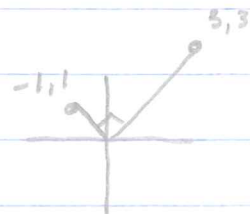
b) $u \cdot v = 2 \cdot (-1) + 4 \cdot 1 = -2 + 4 = 2$

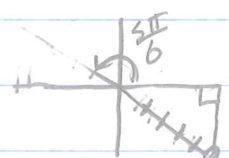
c) $\text{proj}_v(u) = \frac{u \cdot v}{|v|^2} \cdot v = \frac{2}{\sqrt{(-1)^2 + 1^2}^2} v = \frac{2}{2} \cdot v$

$$1 \cdot \langle -1, 1 \rangle = \langle -1, 1 \rangle$$

d) $(u+v) \cdot (u-v)$
 $\langle -1, 5 \rangle \cdot \langle 3, 3 \rangle = -3 + 15 = 12$

e) $u_1 = \text{proj}_v(u) = \langle -1, 1 \rangle$
 $u_2 = u - \text{proj}_v(u) = \langle 3, 3 \rangle$



14) $P(-5, \frac{5\pi}{6})$  $x = 5 \cos \frac{5\pi}{6} = -5 \cdot \frac{\sqrt{3}}{2} = -\frac{5\sqrt{3}}{2}$
 $y = -5 \sin \frac{5\pi}{6} = -5 \cdot \frac{1}{2} = -\frac{5}{2}$
 $(-\frac{5\sqrt{3}}{2}, -\frac{5}{2})$

15) $r = 5 \cos \theta$

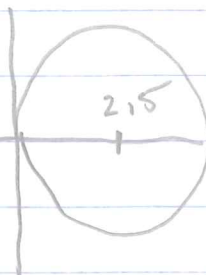
$$r^2 = 5r \cos \theta$$

$$x^2 + y^2 = 5x$$

$$x^2 - 5x + y^2 = 0$$

$$x^2 - 5x + 2.5^2 + y^2 = 2.5^2$$

$$(x - 2.5)^2 + y^2 = 2.5^2$$





$$16) \quad z_1 = -4\sqrt{3} + 4i \quad z_2 = 1 - i$$

$$r = |z_1| = \sqrt{(-4\sqrt{3})^2 + 4^2} = \sqrt{64} = 8$$

$$\tan \theta = \frac{4}{-4\sqrt{3}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\theta = -\frac{\pi}{6} \rightarrow \frac{5\pi}{6}$$

$$z_1 = 8 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$$

$$r = |z_2| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\tan \theta = -1 \quad \theta = -\frac{\pi}{4} = \frac{7\pi}{4}$$

$$z_2 = \sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$$

$$a) \quad z_1 z_2 = 8\sqrt{2} \left(\cos \frac{31\pi}{12} + i \sin \frac{31\pi}{12} \right)$$

$$= 8\sqrt{2} \left(\cos \frac{7\pi}{12} + i \sin \frac{7\pi}{12} \right)$$

$$b) \quad \frac{z_1}{z_2} = \frac{8}{\sqrt{2}} \left(\cos -\frac{11\pi}{12} + i \sin -\frac{11\pi}{12} \right)$$

$$= 4\sqrt{2} \left(\cos \frac{13\pi}{12} + i \sin \frac{13\pi}{12} \right)$$

$$17) \quad (2+2i)^4 \quad r = \sqrt{2^2 + 2^2} = \sqrt{8} \quad \tan \theta = \frac{2}{2} = 1$$

$$\theta = \frac{\pi}{4}$$

$$z = \sqrt{8} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

$$z^4 = \sqrt{8}^4 \left(\cos \frac{4\pi}{4} + i \sin \frac{4\pi}{4} \right)$$

$$= 64 \left(\cos \pi + i \sin \pi \right) = 64(-1 + 0i)$$

$$= -64$$

18) 5th roots of 1 $1+0i$ $r=1$

$\theta=0$

$$z = 1(\cos 0 + i \sin 0)$$

$$z^{1/5} = 1^{1/5} \left(\cos \frac{0+2k\pi}{5} + i \sin \frac{0+2k\pi}{5} \right)$$

$$w_k, k=0 = 1^{1/5} \left(\cos \left(\frac{0}{5} \right) + i \sin \left(\frac{0}{5} \right) \right) = 1$$

$$k=1 = 1 \left(\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5} \right)$$

$$k=2 = 1 \left(\cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5} \right)$$

$$k=3 = 1 \left(\cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5} \right)$$

$$k=4 = 1 \left(\cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5} \right)$$