

Eastern Oregon University  
Concurrent Enrollment/Credit by Proficiency Program

Math 112, Spring, 2016

Final Exam

name/school:

Key

Show any relevant work. For each problem, circle your answer

---

1. (12 points) Solve the triangle below.

$$a = 7, b = 10, c = 7$$

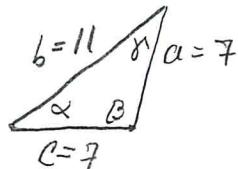
Note that  $\alpha = \beta$ , since triangle is isosceles

$$a^2 = b^2 + c^2 - 2bc \cdot \cos \alpha$$

$$7^2 = 10^2 + 7^2 - 2 \cdot 10 \cdot 7 \cdot \cos \alpha$$

$$140 \cdot \cos \alpha = 100$$

$$\alpha = \cos^{-1} \left( \frac{100}{140} \right) \approx 44.4^\circ$$



$$\theta = 44.4^\circ$$

$$\beta = 180^\circ - 2(44.4^\circ) = 91.2^\circ$$

2. (20 points) For a and b below, write  $z$  into polar form with argument  $\theta$  between 0 and  $2\pi$ .

a)  $z_1 = 1 + \sqrt{3}i$

$$r = |z_1| = \sqrt{1+3} = 2$$

$$\theta = \tan^{-1} \left( \frac{\sqrt{3}}{1} \right)$$

$$= \frac{\pi}{3}$$

$$z_1 = 2 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

b)  $z_2 = -1 + i$

$$r = |z_2| = \sqrt{1+1} = \sqrt{2}$$

$$\tan \theta = -1$$

$$\Rightarrow \theta = \frac{3\pi}{4}$$

$$z_2 = \sqrt{2} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$

- c) Calculate the product  $z_1 z_2$  for  $z_1$  and  $z_2$  above, and find the modulus of the product.

$$z_1 \cdot z_2 = (1 + \sqrt{3}i)(-1 + i) = -1 + i - \sqrt{3}i - \sqrt{3}$$

$$= (-1 - \sqrt{3}) + (1 - \sqrt{3})i$$

$$(-1 - \sqrt{3})^2 = 1 + 2\sqrt{3} + 3$$

$$(1 - \sqrt{3})^2 = 1 - 2\sqrt{3} + 3$$

$$|z_1 \cdot z_2| = \sqrt{(-1 - \sqrt{3})^2 + (1 - \sqrt{3})^2}$$

$$= \sqrt{4+4} = \sqrt{8}$$

- d) For  $z_2$  in part b, calculate  $(z_2)^8$

$$z_2^8 = (\sqrt{2})^8 \left[ \cos \frac{8 \cdot 3\pi}{4} + i \cdot \sin \frac{8 \cdot 3\pi}{4} \right]$$

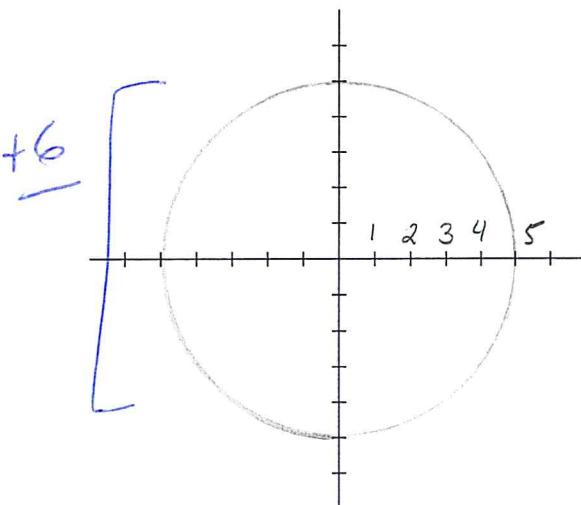
$$= 2^4 [\cos 6\pi + i \cdot \sin 6\pi]$$

$$= 16 [1 + 0] = 16, \text{ or } 16 + 0i$$

3. (12 points) Sketch the graph of the polar curve  $r = 5$ , and convert the equation to rectangular form.

$$\begin{aligned} r^2 &= x^2 + y^2 \\ 25 &= x^2 + y^2 \end{aligned}$$

$\boxed{+6}$



4. (24 points) For each of the following, construct any appropriate reference triangles, and find exact values.

6 a.  $\cos^{-1}(-\frac{1}{2}) = y$

$$\cos y = -\frac{1}{2}$$

$\boxed{+2}$

$$y = \frac{2\pi}{3}$$

$\boxed{+2}$



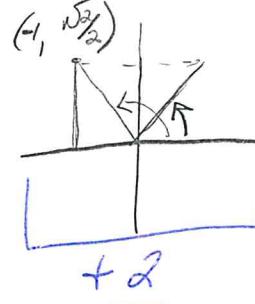
8 b.  $\sin^{-1}(\sin(3\pi/4))$

$$\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$$

$\boxed{-1}$

$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

$\boxed{-1}$



$$-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$$

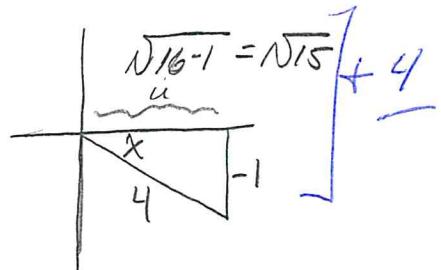
10 c. If  $\sin x = -1/4$  and  $\tan x < 0$ , find the values of the other five elementary trigonometric functions at  $x$ .

$$\sin x = -\frac{1}{4} \Rightarrow \csc x = -4$$

$\boxed{+2}$

$$\left[ \cos x = \frac{\sqrt{15}}{4}, \sec x = \frac{4}{\sqrt{15}} \right]$$

$\boxed{+2}$



$$\left[ \tan x = -\frac{1}{\sqrt{15}}, \cot x = -\sqrt{15} \right]$$

$\boxed{+2}$

$$\begin{aligned} u^2 + 1 &= 16 \\ u^2 &= 15 \end{aligned}$$

5. (10 points) Given the point with polar coordinates  $(-2, \pi/3)$ , convert to rectangular coordinates.

$$x = r \cos \theta = -2 \cos \frac{\pi}{3} = -2 \left(\frac{1}{2}\right) = -1 \quad \boxed{+5} \rightarrow (-1, -\sqrt{3})$$

$$y = r \sin \theta = -2 \sin \frac{\pi}{3} = -2 \left(\frac{\sqrt{3}}{2}\right) = -\sqrt{3} \quad \boxed{+5}$$

6. (10 points) For the following function, sketch one period of the graph carefully, and label the grid sufficiently to indicate the period and either amplitude or asymptotes.

$$f(x) = \frac{3}{2} \sin 3x$$

amplitude =  $\frac{3}{2} \boxed{+2}$

period =  $\frac{2\pi}{3} \boxed{+3}$

$f(0) = 0$

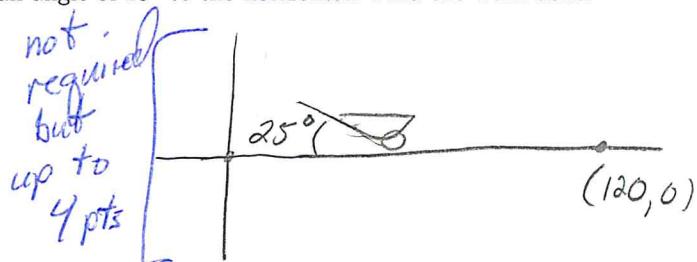
$f\left(\frac{\pi}{6}\right) = \frac{3}{2} \sin \frac{3\pi}{6} = \frac{3}{2} \cdot 1 = \frac{3}{2} \quad \text{be identifiable from graph.}$

etc.

need only

7. (12 points) A wheelbarrow is pushed 120 feet along a horizontal path by a constant force of 75 pounds. The handles of the wheelbarrow are held at an angle of  $25^\circ$  to the horizontal. Find the work done.

$$D = 120 \vec{i} \quad \boxed{+4}$$



$$\boxed{+4} \quad F = 75 \cos 25^\circ \vec{i} + 75 \sin 25^\circ \vec{j}$$

$$\boxed{+4} \quad W = F \cdot D = (75 \cos 25^\circ) 120 + (75 \sin 25^\circ) \cdot 0$$

$$\approx 8157 \text{ ft-lb. units required, -2 if missing}$$