

Math 112: Exam 3 Review

I took a quick look at the problems in the review and most seem applicable; except the calculator identified ones (#25-30, 47, 48, 49, 72). Problems #31-46 we found all the solutions, not just the ones $[0, 2\pi)$. Know the difference and be ready to find all of them since the professor assigned us only that kind. Additionally, I don't remember doing a problem like #49, although I think we could. I accidentally skipped a problem like #50 in my solutions – although if you ask me I'll tell you why it's easy.

In the Chapter 9 review, questions #1-10 would apply to what we learned and test questions #1-3.

Below is the correspondence I've had with the professor regarding the test. Be sure you note that the notes are restricted to one half page (yes, only one side), and also restricted as to what formulas you can put on it. Remember, you will need to turn in your formula sheet with your test.

All,

As you prepare for exam 3, the students may use one half-page of notes to record the various identities. Any of the various formulas contained in the blue boxes are allowed on the note page. There will be a few verifications, but most use of identities will be in evaluation and solving of equations. Students should submit their note pages with their exams.

Hi Pete,

For verifying, they should refer to any other basic identities they use and substitutions they make.

I would encourage them to include any variable names in identities. It should help them remember that these are functions, which is valuable in calculus where domains matter, and it might be important to some future instructor.

Hi Pete,

No, there aren't any applied vector problems this year. For verifying identities: students should start with one side of the stated equation, usually the more complicated looking side. They will often make some substitution, such as replacing tangent with sine/cosine, or they may see $\sin^2(x) + \cos^2(x)$ and replace it with 1 because of the pythagorean identity. Justifying a step just means a quick explanation of the substitution they are making.

hope it helps,

CHAPTER 7 | REVIEW

CONCEPT CHECK

- (a) State the reciprocal identities.
(b) State the Pythagorean identities.
(c) State the even-odd identities.
(d) State the cofunction identities.
- Explain the difference between an equation and an identity.
- How do you prove a trigonometric identity?
- (a) State the Addition Formulas for Sine, Cosine, and Tangent.
(b) State the Subtraction Formulas for Sine, Cosine, and Tangent.
- (a) State the Double-Angle Formulas for Sine, Cosine, and Tangent.
(b) State the formulas for lowering powers.
(c) State the Half-Angle Formulas.
- (a) State the Product-to-Sum Formulas.
(b) State the Sum-to-Product Formulas.
- Explain how you solve a trigonometric equation by factoring.
- What identity would you use to solve the equation $\cos x - \sin 2x = 0$?

EXERCISES

1–24 ■ Verify the identity.

- $\sin \theta (\cot \theta + \tan \theta) = \sec \theta$
- $(\sec \theta - 1)(\sec \theta + 1) = \tan^2 \theta$
- $\cos^2 x \csc x - \csc x = -\sin x$
- $\frac{1}{1 - \sin^2 x} = 1 + \tan^2 x$
- $\frac{\cos^2 x - \tan^2 x}{\sin^2 x} = \cot^2 x - \sec^2 x$
- $\frac{1 + \sec x}{\sec x} = \frac{\sin^2 x}{1 - \cos x}$
- $\frac{\cos^2 x}{1 - \sin x} = \frac{\cos x}{\sec x - \tan x}$
- $(1 - \tan x)(1 - \cot x) = 2 - \sec x \csc x$
- $\sin^2 x \cot^2 x + \cos^2 x \tan^2 x = 1$
- $(\tan x + \cot x)^2 = \csc^2 x \sec^2 x$
- $\frac{\sin 2x}{1 + \cos 2x} = \tan x$
- $\frac{\cos(x + y)}{\cos x \sin y} = \cot y - \tan x$
- $\tan \frac{x}{2} = \csc x - \cot x$
- $\frac{\sin(x + y) + \sin(x - y)}{\cos(x + y) + \cos(x - y)} = \tan x$
- $\sin(x + y) \sin(x - y) = \sin^2 x - \sin^2 y$
- $\csc x - \tan \frac{x}{2} = \cot x$
- $1 + \tan x \tan \frac{x}{2} = \sec x$
- $\frac{\sin 3x + \cos 3x}{\cos x - \sin x} = 1 + 2 \sin 2x$

19. $\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)^2 = 1 - \sin x$

20. $\frac{\cos 3x - \cos 7x}{\sin 3x + \sin 7x} = \tan 2x$

21. $\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = \sec x$

22. $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 2 + 2 \cos(x + y)$

23. $\tan\left(x + \frac{\pi}{4}\right) = \frac{1 + \tan x}{1 - \tan x}$

24. $\frac{\sec x - 1}{\sin x \sec x} = \tan \frac{x}{2}$

25–28 ■ (a) Graph f and g . (b) Do the graphs suggest that the equation $f(x) = g(x)$ is an identity? Prove your answer.

25. $f(x) = 1 - \left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)^2$, $g(x) = \sin x$

26. $f(x) = \sin x + \cos x$, $g(x) = \sqrt{\sin^2 x + \cos^2 x}$

27. $f(x) = \tan x \tan \frac{x}{2}$, $g(x) = \frac{1}{\cos x}$

28. $f(x) = 1 - 8 \sin^2 x + 8 \sin^4 x$, $g(x) = \cos 4x$

29–30 ■ (a) Graph the function(s) and make a conjecture, and (b) prove your conjecture.

29. $f(x) = 2 \sin^2 3x + \cos 6x$

30. $f(x) = \sin x \cot \frac{x}{2}$, $g(x) = \cos x$

31–48 ■ Solve the equation in the interval $[0, 2\pi)$.

31. $4 \sin \theta - 3 = 0$

32. $5 \cos \theta + 3 = 0$

33. $\cos x \sin x - \sin x = 0$

34. $\sin x - 2 \sin^2 x = 0$

35. $2 \sin^2 x - 5 \sin x + 2 = 0$

36. $\sin x - \cos x - \tan x = -1$

37. $2 \cos^2 x - 7 \cos x + 3 = 0$

38. $4 \sin^2 x + 2 \cos^2 x = 3$

39. $\frac{1 - \cos x}{1 + \cos x} = 3$

40. $\sin x = \cos 2x$

41. $\tan^3 x + \tan^2 x - 3 \tan x - 3 = 0$

42. $\cos 2x \csc^2 x = 2 \cos 2x$

43. $\tan \frac{1}{2} x + 2 \sin 2x = \csc x$

44. $\cos 3x + \cos 2x + \cos x = 0$

45. $\tan x + \sec x = \sqrt{3}$

46. $2 \cos x - 3 \tan x = 0$

47. $\cos x = x^2 - 1$

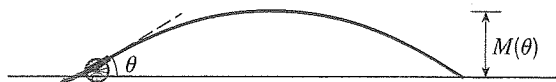
48. $e^{\sin x} = x$

49. If a projectile is fired with velocity v_0 at an angle θ , then the maximum height it reaches (in feet) is modeled by the function

$$M(\theta) = \frac{v_0^2 \sin^2 \theta}{64}$$

Suppose $v_0 = 400$ ft/s.

- (a) At what angle θ should the projectile be fired so that the maximum height it reaches is 2000 ft?
 (b) Is it possible for the projectile to reach a height of 3000 ft?
 (c) Find the angle θ for which the projectile will travel highest.



50. The displacement of an automobile shock absorber is modeled by the function

$$f(t) = 2^{-0.2t} \sin 4\pi t$$

Find the times when the shock absorber is at its equilibrium position (that is, when $f(t) = 0$). [Hint: $2^x > 0$ for all real x .]

- 51–60 ■ Find the exact value of the expression.

51. $\cos 15^\circ$

52. $\sin \frac{5\pi}{12}$

53. $\tan \frac{\pi}{8}$

54. $2 \sin \frac{\pi}{12} \cos \frac{\pi}{12}$

55. $\sin 5^\circ \cos 40^\circ + \cos 5^\circ \sin 40^\circ$

56. $\frac{\tan 66^\circ - \tan 6^\circ}{1 + \tan 66^\circ \tan 6^\circ}$

57. $\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8}$

58. $\frac{1}{2} \cos \frac{\pi}{12} + \frac{\sqrt{3}}{2} \sin \frac{\pi}{12}$

59. $\cos 37.5^\circ \cos 7.5^\circ$

60. $\cos 67.5^\circ + \cos 22.5^\circ$

- 61–66 ■ Find the exact value of the expression given that $\sec x = \frac{3}{2}$, $\csc y = 3$, and x and y are in Quadrant I.

61. $\sin(x + y)$

62. $\cos(x - y)$

63. $\tan(x + y)$

64. $\sin 2x$

65. $\cos \frac{y}{2}$

66. $\tan \frac{y}{2}$

- 67–68 ■ Find the exact value of the expression.

67. $\tan(2 \cos^{-1} \frac{3}{7})$

68. $\sin(\tan^{-1} \frac{3}{4} + \cos^{-1} \frac{5}{13})$

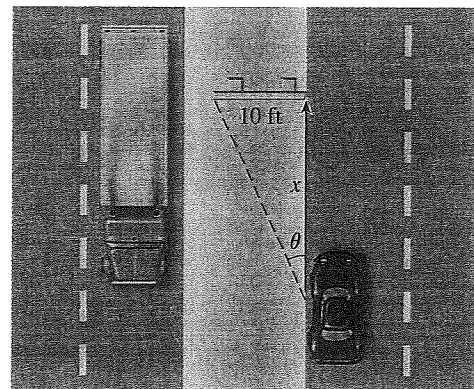
- 69–70 ■ Write the expression as an algebraic expression in the variable(s).

69. $\tan(2 \tan^{-1} x)$

70. $\cos(\sin^{-1} x + \cos^{-1} y)$

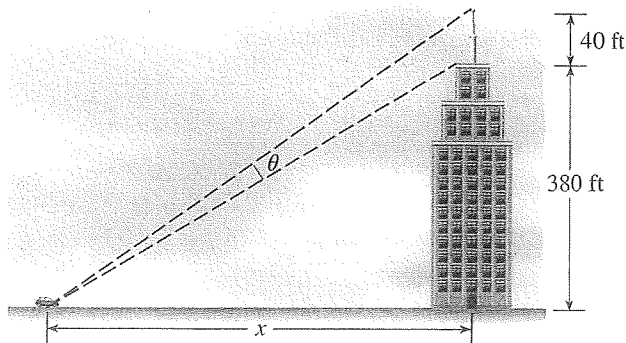
71. A 10-ft-wide highway sign is adjacent to a roadway, as shown in the figure. As a driver approaches the sign, the viewing angle θ changes.

- (a) Express viewing angle θ as a function of the distance x between the driver and the sign.
 (b) The sign is legible when the viewing angle is 2° or greater. At what distance x does the sign first become legible?



72. A 380-ft-tall building supports a 40-ft communications tower (see the figure). As a driver approaches the building, the viewing angle θ of the tower changes.

- (a) Express the viewing angle θ as a function of the distance x between the driver and the building.
 (b) At what distance from the building is the viewing angle θ as large as possible?



1. Verify each identity.

(a) $\tan \theta \sin \theta + \cos \theta = \sec \theta$

(b) $\frac{\tan x}{1 - \cos x} = \csc x (1 + \sec x)$

(c) $\frac{2 \tan x}{1 + \tan^2 x} = \sin 2x$

2. Let $x = 2 \sin \theta$, $-\pi/2 < \theta < \pi/2$. Simplify the expression

$$\frac{x}{\sqrt{4 - x^2}}$$

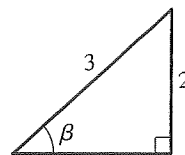
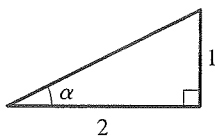
3. Find the exact value of each expression.

(a) $\sin 8^\circ \cos 22^\circ + \cos 8^\circ \sin 22^\circ$

(b) $\sin 75^\circ$

(c) $\sin \frac{\pi}{12}$

4. For the angles α and β in the figures, find $\cos(\alpha + \beta)$.



5. (a) Write $\sin 3x \cos 5x$ as a sum of trigonometric functions.

(b) Write $\sin 2x - \sin 5x$ as a product of trigonometric functions.

6. If $\sin \theta = -\frac{4}{5}$ and θ is in Quadrant III, find $\tan(\theta/2)$.

7. Solve each trigonometric equation in the interval $[0, 2\pi)$, rounded to two decimal places.

(a) $3 \sin \theta - 1 = 0$

(b) $(2 \cos \theta - 1)(\sin \theta - 1) = 0$

(c) $2 \cos^2 \theta + 5 \cos \theta + 2 = 0$

(d) $\sin 2\theta - \cos \theta = 0$

8. Find all solutions in the interval $[0, 2\pi)$, rounded to five decimal places:

$$5 \cos 2\theta = 2$$

9. Find the exact value of $\cos(2 \tan^{-1} \frac{9}{40})$.

10. Rewrite the expression as an algebraic function of x and y :

$$\sin(\cos^{-1} x - \tan^{-1} y)$$

6. Express the dot product $\mathbf{u} \cdot \mathbf{v}$ in terms of the components of the vectors.
- (a) $\mathbf{u} = \langle u_1, u_2 \rangle, \mathbf{v} = \langle v_1, v_2 \rangle$
 (b) $\mathbf{u} = \langle u_1, u_2, u_3 \rangle, \mathbf{v} = \langle v_1, v_2, v_3 \rangle$
7. (a) How do you use the dot product to find the angle between two vectors?
 (b) How do you use the dot product to determine whether two vectors are perpendicular?
8. What is the component of \mathbf{u} along \mathbf{v} , and how do you calculate it?
9. What is the projection of \mathbf{u} onto \mathbf{v} , and how do you calculate it?
10. How much work is done by the force \mathbf{F} in moving an object along a displacement \mathbf{D} ?
11. How do you find the distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in three-dimensional space?
12. What is the equation of the sphere with center $C(a, b, c)$ and radius r ?
13. (a) How do you calculate the cross product $\mathbf{a} \times \mathbf{b}$ if you know the components of \mathbf{a} and \mathbf{b} ?
 (b) How do you calculate $\mathbf{a} \times \mathbf{b}$ if you know the lengths of \mathbf{a} and \mathbf{b} and the angle between them?
 (c) What is the angle between $\mathbf{a} \times \mathbf{b}$ and each of \mathbf{a} and \mathbf{b} ?
14. (a) How do you find the area of the parallelogram determined by \mathbf{a} and \mathbf{b} ?
 (b) How do you find the volume of the parallelepiped determined by \mathbf{a} , \mathbf{b} , and \mathbf{c} ?
15. Write parametric equations for the line that contains the point $P(x_0, y_0, z_0)$ and that is parallel to the vector $\mathbf{v} = \langle a, b, c \rangle$.
16. Write an equation for the plane that contains the point $P(x_0, y_0, z_0)$ and has normal vector $\mathbf{n} = \langle a, b, c \rangle$.
17. How do you find parametric equations for the line that contains the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$?
18. How do you find an equation for the plane that contains the points $P(x_1, y_1, z_1)$, $Q(x_2, y_2, z_2)$, and $R(x_3, y_3, z_3)$?

EXERCISES

Exercises 1–24 deal with vectors in two dimensions.

1–4 ■ Find $|\mathbf{u}|$, $\mathbf{u} + \mathbf{v}$, $\mathbf{u} - \mathbf{v}$, $2\mathbf{u}$, and $3\mathbf{u} - 2\mathbf{v}$.

1. $\mathbf{u} = \langle -2, 3 \rangle, \mathbf{v} = \langle 8, 1 \rangle$

2. $\mathbf{u} = \langle 5, -2 \rangle, \mathbf{v} = \langle -3, 0 \rangle$

3. $\mathbf{u} = 2\mathbf{i} + \mathbf{j}, \mathbf{v} = \mathbf{i} - 2\mathbf{j}$

4. $\mathbf{u} = 3\mathbf{j}, \mathbf{v} = -\mathbf{i} + 2\mathbf{j}$

5. Find the vector with initial point $P(0, 3)$ and terminal point $Q(3, -1)$.

6. If the vector $5\mathbf{i} - 8\mathbf{j}$ is placed in the plane with its initial point at $P(5, 6)$, find its terminal point.

7–8 ■ Find the length and direction of the given vector.

7. $\mathbf{u} = \langle -2, 2\sqrt{3} \rangle$

8. $\mathbf{v} = 2\mathbf{i} - 5\mathbf{j}$

9–10 ■ The length $|\mathbf{u}|$ and direction θ of a vector \mathbf{u} are given. Express \mathbf{u} in component form.

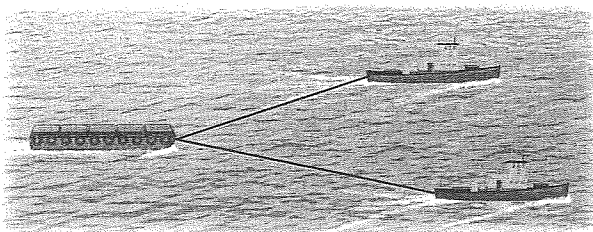
9. $|\mathbf{u}| = 20, \theta = 60^\circ$

10. $|\mathbf{u}| = 13.5, \theta = 125^\circ$

11. Two tugboats are pulling a barge as shown in the figure. One pulls with a force of 2.0×10^4 lb in the direction N 50° E, and the other pulls with a force of 3.4×10^4 lb in the direction S 75° E.

(a) Find the resultant force on the barge as a vector.

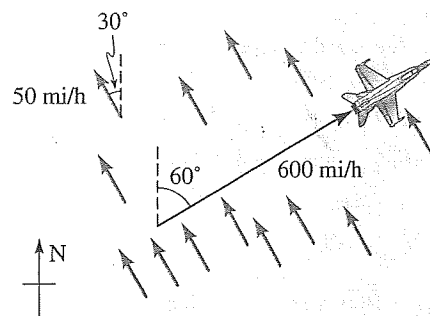
(b) Find the magnitude and direction of the resultant force.



12. An airplane heads N 60° E at a speed of 600 mi/h relative to the air. A wind begins to blow in the direction N 30° W at 50 mi/h. (See the figure.)

(a) Find the velocity of the airplane as a vector.

(b) Find the true speed and direction of the airplane.



13–16 ■ Find the vectors $|\mathbf{u}|$, $\mathbf{u} \cdot \mathbf{u}$, and $\mathbf{u} \cdot \mathbf{v}$.

13. $\mathbf{u} = \langle 4, -3 \rangle, \mathbf{v} = \langle 9, -8 \rangle$

14. $\mathbf{u} = \langle 5, 12 \rangle, \mathbf{v} = \langle 10, -4 \rangle$

15. $\mathbf{u} = -2\mathbf{i} + 2\mathbf{j}, \mathbf{v} = \mathbf{i} + \mathbf{j}$

16. $\mathbf{u} = 10\mathbf{j}, \mathbf{v} = 5\mathbf{i} - 3\mathbf{j}$

17–20 ■ Are \mathbf{u} and \mathbf{v} orthogonal? If not, find the angle between them.

17. $\mathbf{u} = \langle -4, 2 \rangle, \mathbf{v} = \langle 3, 6 \rangle$

18. $\mathbf{u} = \langle 5, 3 \rangle, \mathbf{v} = \langle -2, 6 \rangle$

19. $\mathbf{u} = 2\mathbf{i} + \mathbf{j}, \mathbf{v} = \mathbf{i} + 3\mathbf{j}$

20. $\mathbf{u} = \mathbf{i} - \mathbf{j}, \mathbf{v} = \mathbf{i} + \mathbf{j}$

- Let \mathbf{u} be the vector with initial point $P(3, -1)$ and terminal point $Q(-3, 9)$.
 - Graph \mathbf{u} in the coordinate plane.
 - Express \mathbf{u} in terms of \mathbf{i} and \mathbf{j} .
 - Find the length of \mathbf{u} .
- Let $\mathbf{u} = \langle 1, 3 \rangle$ and $\mathbf{v} = \langle -6, 2 \rangle$.
 - Find $\mathbf{u} - 3\mathbf{v}$.
 - Find $|\mathbf{u} + \mathbf{v}|$.
 - Find $\mathbf{u} \cdot \mathbf{v}$.
 - Are \mathbf{u} and \mathbf{v} perpendicular?
- Let $\mathbf{u} = \langle -4\sqrt{3}, 4 \rangle$.
 - Graph \mathbf{u} in the coordinate plane, with initial point $(0, 0)$.
 - Find the length and direction of \mathbf{u} .
- A river is flowing due east at 8 mi/h. A man heads his motorboat in the direction $N 30^\circ E$ in the river. The speed of the motorboat relative to the water is 12 mi/h.
 - Express the true velocity of the motorboat as a vector.
 - Find the true speed and direction of the motorboat.
- Let $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{v} = 5\mathbf{i} - \mathbf{j}$.
 - Find the angle between \mathbf{u} and \mathbf{v} .
 - Find the component of \mathbf{u} along \mathbf{v} .
 - Find $\text{proj}_{\mathbf{v}} \mathbf{u}$.
- Find the work done by the force $\mathbf{F} = 3\mathbf{i} - 5\mathbf{j}$ in moving an object from the point $(2, 2)$ to the point $(7, -13)$.
- Let $P(4, 3, -1)$ and $Q(6, -1, 3)$ be two points in three-dimensional space.
 - Find the distance between P and Q .
 - Find an equation for the sphere whose center is P and for which the segment \overline{PQ} is a radius of the sphere.
 - The vector \mathbf{u} has initial point P and terminal point Q . Express \mathbf{u} both in component form and using the vectors \mathbf{i} , \mathbf{j} , and \mathbf{k} .
- Calculate the given quantity if

$$\mathbf{a} = \mathbf{i} + \mathbf{j} - 2\mathbf{k} \quad \mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k} \quad \mathbf{c} = \mathbf{j} - 5\mathbf{k}$$
 - $2\mathbf{a} + 3\mathbf{b}$
 - $|\mathbf{a}|$
 - $\mathbf{a} \cdot \mathbf{b}$
 - $\mathbf{a} \times \mathbf{b}$
 - $|\mathbf{b} \times \mathbf{c}|$
 - $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$
 - The angle between \mathbf{a} and \mathbf{b} (rounded to the nearest degree)
- Find two unit vectors that are perpendicular to both $\mathbf{j} + 2\mathbf{k}$ and $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$.
- Find a vector perpendicular to the plane that contains the points $P(1, 0, 0)$, $Q(2, 0, -1)$, and $R(1, 4, 3)$.
 - Find an equation for the plane that contains P , Q , and R .
 - Find the area of triangle PQR .
- Find parametric equations for the line that contains the points $P(2, -4, 7)$ and $Q(0, -3, 5)$.

17. (a) $\frac{\pi}{9} + \frac{2k\pi}{3}, \frac{5\pi}{9} + \frac{2k\pi}{3}$ (b) $\pi/9, 5\pi/9, 7\pi/9, 11\pi/9,$

$13\pi/9, 17\pi/9$ 18. (a) $\frac{\pi}{3} + k\pi, \frac{2\pi}{3} + k\pi$

(b) $\pi/3, 2\pi/3, 4\pi/3, 5\pi/3$ 19. (a) $\frac{\pi}{3} + k\pi, \frac{2\pi}{3} + k\pi$

(b) $\pi/3, 2\pi/3, 4\pi/3, 5\pi/3$ 20. (a) $\frac{7\pi}{18} + \frac{2k\pi}{3}, \frac{11\pi}{18} + \frac{2k\pi}{3}$

(b) $7\pi/18, 11\pi/18, 19\pi/18, 23\pi/18, 31\pi/18, 35\pi/18$

21. (a) $\frac{5\pi}{18} + \frac{k\pi}{3}$ (b) $5\pi/18, 11\pi/18, 17\pi/18, 23\pi/18,$

$29\pi/18, 35\pi/18$ 22. (a) $\frac{\pi}{12} + \frac{k\pi}{2}, \frac{5\pi}{12} + \frac{k\pi}{2}$

(b) $\pi/12, 5\pi/12, 7\pi/12, 11\pi/12, 13\pi/12, 17\pi/12, 19\pi/12, 23\pi/12$

23. (a) $4k\pi$ (b) 0 24. (a) $\frac{8\pi}{3} + 4k\pi$ (b) None

25. (a) $4\pi + 6k\pi, 5\pi + 6k\pi$ (b) None 26. (a) $2k\pi$ (b) 0

27. (a) $0.62 + \frac{k\pi}{2}$ (b) 0.62, 2.19, 3.76, 5.33

28. (a) $0.15 + \frac{k\pi}{3}, 0.89 + \frac{k\pi}{3}$ (b) 0.15, 0.89, 1.20, 1.94, 2.24,

2.98, 3.29, 4.03, 4.34, 5.08, 5.39, 6.13 29. (a) $k\pi$ (b) 0, π

30. (a) $2k\pi/3$ (b) 0, $2\pi/3, 4\pi/3$

31. (a) $\frac{\pi}{6} + k\pi, \frac{\pi}{4} + k\pi, \frac{5\pi}{6} + k\pi$

(b) $\pi/6, \pi/4, 5\pi/6, 7\pi/6, 5\pi/4, 11\pi/6$

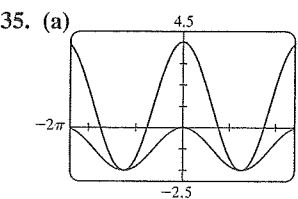
32. (a) $\frac{\pi}{6} + 2k\pi, \frac{2\pi}{3} + 2k\pi, \frac{5\pi}{6} + 2k\pi, \frac{4\pi}{3} + 2k\pi$

(b) $\pi/6, 2\pi/3, 5\pi/6, 4\pi/3$

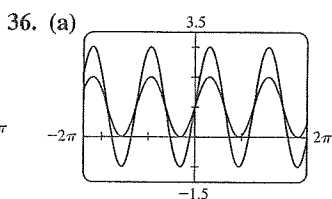
33. (a) $\frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi, \frac{3\pi}{4} + k\pi$

(b) $\pi/6, 3\pi/4, 5\pi/6, 7\pi/4$

34. (a) $\frac{\pi}{4} + k\pi, \frac{3\pi}{4} + k\pi$ (b) $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$



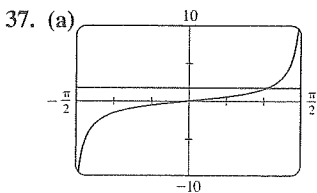
$(\pm 3.14, -2)$



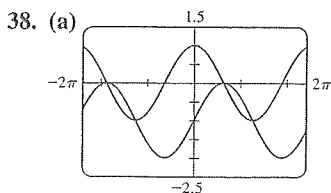
$(\pm 6.28, 1), (\pm 4.71, 1),$
 $(\pm 3.14, 1), (\pm 1.57, 1), (0, 1)$

(b) $((2k + 1)\pi, -2)$

(b) $(\frac{1}{2}k\pi, 1)$



$(1.04, 1.73)$



$(-4.71, 0), (-3.14, 1),$
 $(1.57, 0), (3.14, -1)$

(b) $(\frac{\pi}{3} + k\pi, \sqrt{3})$

(b) $(\pi + 2k\pi, -1), (\frac{\pi}{2} + 2k\pi, 0)$

39. $\pi/8, 3\pi/8, 5\pi/8, 7\pi/8, 9\pi/8, 11\pi/8, 13\pi/8, 15\pi/8$

40. $\pi/3, 5\pi/3$ 41. $\pi/3, 2\pi/3$ 42. 0, $\pi/2, \pi, 3\pi/2$

43. $\pi/2, 7\pi/6, 3\pi/2, 11\pi/6$ 44. 0, $\pi/2, 3\pi/2$ 45. 0

46. $\pi/8, 3\pi/8, 5\pi/8, 7\pi/8, 9\pi/8, 11\pi/8, 13\pi/8, 15\pi/8$

47. 0, π 48. $\pi/3, 2\pi/3, 4\pi/3, 5\pi/3$

49. 0, $\pi/3, 2\pi/3, \pi, 4\pi/3, 5\pi/3$ 50. 0, $\pi/9, \pi/3, 5\pi/9, 2\pi/3,$

$7\pi/9, \pi, 11\pi/9, 4\pi/3, 13\pi/9, 5\pi/3, 17\pi/9$ 51. $\pi/6, 3\pi/2$

52. 1.15, 3.57 53. $k\pi/2$ 54. $k\pi/6$

55. $\frac{\pi}{2} + k\pi, \frac{\pi}{9} + \frac{2k\pi}{3}, \frac{5\pi}{9} + \frac{2k\pi}{3}$

56. $\frac{\pi}{8} + \frac{k\pi}{4}, \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$ 57. 0, ± 0.95

58. 1.17, -2.66, -2.94 59. 1.92 60. 0, ± 0.93 61. ± 0.71

62. 0 63. 0.94721° or 89.05279° 64. $0, \frac{1}{2}, 1, \frac{3}{2}, \dots$

65. (a) 34th day (February 3), 308th day (November 4)

(b) 275 days 66. (b) $1.047 \text{ rad} \approx 60^\circ$

CHAPTER 7 REVIEW ■ PAGE 530

1. LHS = $\sin \theta \left(\frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \right) = \cos \theta + \frac{\sin^2 \theta}{\cos \theta}$
= $\frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta} = \text{RHS}$

2. LHS = $\sec^2 \theta - 1 = \text{RHS}$

3. LHS = $(1 - \sin^2 x) \csc x - \csc x$
= $\csc x - \sin^2 x \csc x - \csc x$
= $-\sin^2 x \frac{1}{\sin x} = \text{RHS}$

4. LHS = $\frac{1}{\cos^2 x} = \sec^2 x = \text{RHS}$

5. LHS = $\frac{\cos^2 x}{\sin^2 x} - \frac{\tan^2 x}{\sin^2 x} = \cot^2 x - \frac{1}{\cos^2 x} = \text{RHS}$

6. LHS = $\frac{1}{\sec x} + 1 = 1 + \cos x = (1 + \cos x) \cdot \frac{1 - \cos x}{1 - \cos x}$
= $\frac{1 - \cos^2 x}{1 - \cos x} = \text{RHS}$

7. LHS = $\frac{\cos x}{\frac{1}{\cos x}(1 - \sin x)} = \frac{\cos x}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}} = \text{RHS}$

8. LHS = $1 - \cot x - \tan x + \tan x \cot x$
= $2 - (\cot x + \tan x)$
= $2 - \left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} \right) = 2 - \frac{\cos^2 x + \sin^2 x}{\cos x \sin x}$
= $2 - \frac{1}{\cos x \sin x} = \text{RHS}$

9. LHS = $\sin^2 x \frac{\cos^2 x}{\sin^2 x} + \cos^2 x \frac{\sin^2 x}{\cos^2 x} = \cos^2 x + \sin^2 x = \text{RHS}$

10. LHS = $\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)^2 = \left(\frac{\sin^2 x + \cos^2 x}{\cos x \sin x} \right)^2$
= $\left(\frac{1}{\cos x \sin x} \right)^2 = \text{RHS}$

11. LHS = $\frac{2 \sin x \cos x}{1 + 2 \cos^2 x - 1} = \frac{2 \sin x \cos x}{2 \cos^2 x} = \frac{2 \sin x}{2 \cos x} = \text{RHS}$

12. LHS = $\frac{\cos x \cos y - \sin x \sin y}{\cos x \sin y}$
= $\frac{\cos x \cos y}{\cos x \sin y} - \frac{\sin x \sin y}{\cos x \sin y} = \frac{\cos y}{\sin y} - \frac{\sin x}{\cos x} = \text{RHS}$

$$13. \text{LHS} = \frac{1 - \cos x}{\sin x} = \frac{1}{\sin x} - \frac{\cos x}{\sin x} = \text{RHS}$$

$$14. \text{LHS} = \frac{2 \sin\left(\frac{(x+y) + (x-y)}{2}\right) \cos\left(\frac{(x+y) - (x-y)}{2}\right)}{2 \cos\left(\frac{(x+y) + (x-y)}{2}\right) \cos\left(\frac{(x+y) - (x-y)}{2}\right)}$$

$$= \frac{2 \sin x \cos y}{2 \cos x \cos y} = \text{RHS}$$

$$15. \text{LHS} = \frac{1}{2}[\cos((x+y) - (x-y)) - \cos((x+y) + (x-y))]$$

$$= \frac{1}{2}(\cos 2y - \cos 2x)$$

$$= \frac{1}{2}[1 - 2 \sin^2 y - (1 - 2 \sin^2 x)]$$

$$= \frac{1}{2}(2 \sin^2 x - 2 \sin^2 y) = \text{RHS}$$

$$16. \text{LHS} = \csc x - \frac{1 - \cos x}{\sin x}$$

$$= \csc x - (\csc x - \cot x) = \text{RHS}$$

$$17. \text{LHS} = 1 + \frac{\sin x}{\cos x} \cdot \frac{1 - \cos x}{\sin x} = 1 + \frac{1 - \cos x}{\cos x}$$

$$= 1 + \frac{1}{\cos x} - 1 = \text{RHS}$$

$$18. \text{LHS} = \frac{\sin(x+2x) + \cos(x+2x)}{\cos x - \sin x}$$

$$= \frac{\sin x \cos 2x + \cos x \sin 2x + \cos x \cos 2x - \sin x \sin 2x}{\cos x - \sin x}$$

$$= \frac{\cos 2x(\sin x + \cos x) + \sin 2x(\cos x - \sin x)}{\cos x - \sin x}$$

$$= \frac{\cos 2x(\sin x + \cos x)}{\cos x - \sin x} + \sin 2x$$

$$= \frac{(\cos^2 x - \sin^2 x)(\sin x + \cos x)}{\cos x - \sin x} + \sin 2x$$

$$= (\cos x + \sin x)(\sin x + \cos x) + \sin 2x$$

$$= \sin^2 x + \cos^2 x + 2 \sin x \cos x + \sin 2x$$

$$= 1 + \sin 2x + \sin 2x = \text{RHS}$$

$$19. \text{LHS} = \cos^2 \frac{x}{2} - 2 \sin \frac{x}{2} \cos \frac{x}{2} + \sin^2 \frac{x}{2}$$

$$= 1 - \sin(2 \cdot \frac{x}{2}) = \text{RHS}$$

$$20. \text{LHS} = \frac{-2 \sin\left(\frac{3x+7x}{2}\right) \sin\left(\frac{3x-7x}{2}\right)}{2 \sin\left(\frac{3x+7x}{2}\right) \cos\left(\frac{3x-7x}{2}\right)} = \frac{-2 \sin 5x \sin(-2x)}{2 \sin 5x \cos(-2x)}$$

$$= \frac{2 \sin 5x \sin 2x}{2 \sin 5x \cos 2x} = \text{RHS}$$

$$21. \text{LHS} = \frac{2 \sin x \cos x}{\sin x} - \frac{2 \cos^2 x - 1}{\cos x}$$

$$= 2 \cos x - 2 \cos x + \frac{1}{\cos x} = \text{RHS}$$

$$22. \text{LHS} = \cos^2 x + 2 \cos x \cos y + \cos^2 y$$

$$+ \sin^2 x - 2 \sin x \sin y + \sin^2 y$$

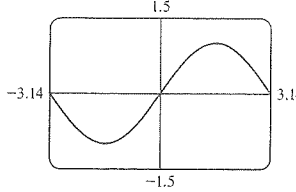
$$= (\cos^2 x + \sin^2 x) + (\sin^2 y + \cos^2 y)$$

$$+ 2(\cos x \cos y - \sin x \sin y) = \text{RHS}$$

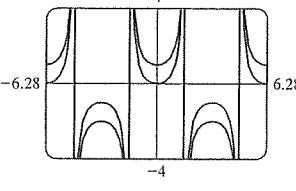
$$23. \text{LHS} = \frac{\tan x + \tan \frac{\pi}{4}}{1 - \tan x \tan \frac{\pi}{4}} = \text{RHS}$$

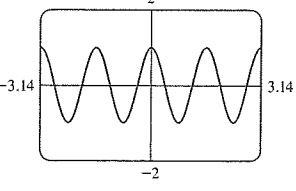
$$24. \text{LHS} = \frac{\frac{1}{\cos x} - 1}{\sin x \frac{1}{\cos x}} = \left(\frac{1}{\cos x} - 1\right) \frac{\cos x}{\sin x}$$

$$= \frac{1 - \cos x}{\sin x} = \text{RHS}$$

25. (a)  (b) Yes

26. (a)  (b) No

27. (a)  (b) No

28. (a)  (b) Yes

29. (a)  $2 \sin^2 3x + \cos 6x = 1$

30. (a)  $\sin x \cot \frac{x}{2} = \cos x + 1$

31. 0.85, 2.29 32. 2.21, 4.07 33. $0, \pi$ 34. $0, \pi/6, 5\pi/6, \pi$
 35. $\pi/6, 5\pi/6$ 36. $0, \pi/4, 5\pi/4$ 37. $\pi/3, 5\pi/3$
 38. $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ 39. $2\pi/3, 4\pi/3$
 40. $\pi/6, 3\pi/2, 5\pi/6$ 41. $\pi/3, 2\pi/3, 3\pi/4, 4\pi/3, 5\pi/3, 7\pi/4$
 42. $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$
 43. $\pi/6, \pi/2, 5\pi/6, 7\pi/6, 3\pi/2, 11\pi/6$
 44. $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4, 2\pi/3, 4\pi/3$ 45. $\pi/6$
 46. $\pi/6, 5\pi/6$ 47. 1.18 48. 2.22 49. (a) 63.4° (b) No
 (c) 90° 50. $t = 0.25k, k = 0, 1, 2, \dots$ 51. $\frac{1}{2}\sqrt{2 + \sqrt{3}}$
 52. $\frac{\sqrt{2} + \sqrt{6}}{4}$ 53. $\sqrt{2} - 1$ 54. $\frac{1}{2}$ 55. $\sqrt{2}/2$
 56. $\sqrt{3}$ 57. $\sqrt{2}/2$ 58. $\sqrt{2}/2$ 59. $\frac{\sqrt{2} + \sqrt{3}}{4}$
 60. $\sqrt{\frac{2 + \sqrt{2}}{2}}$ 61. $2 \frac{\sqrt{10} + 1}{9}$ 62. $\frac{1}{9}(4\sqrt{2} + \sqrt{5})$

63. $\frac{2}{3}(\sqrt{2} + \sqrt{5})$ 64. $4\sqrt{5}/9$ 65. $\sqrt{(3 + 2\sqrt{2})}/6$

66. $3 - 2\sqrt{2}$ 67. $-\frac{12\sqrt{10}}{31}$ 68. $\frac{63}{65}$ 69. $\frac{2x}{1-x^2}$

70. $y\sqrt{1-x^2} - x\sqrt{1-y^2}$

71. (a) $\theta = \tan^{-1}\left(\frac{10}{x}\right)$ (b) 286.4 ft

72. (a) $\theta = \tan^{-1}\left(\frac{420}{x}\right) - \tan^{-1}\left(\frac{380}{x}\right)$ (b) 400 ft

CHAPTER 7 TEST ■ PAGE 532

1. (a) LHS = $\frac{\sin \theta}{\cos \theta} \sin \theta + \cos \theta = \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} = \text{RHS}$

(b) LHS = $\frac{\tan x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} = \frac{\tan x(1 + \cos x)}{1 - \cos^2 x}$
 $= \frac{\frac{\sin x}{\cos x}(1 + \cos x)}{\sin^2 x} = \frac{1}{\sin x} \cdot \frac{1 + \cos x}{\cos x} = \text{RHS}$

(c) LHS = $\frac{2 \tan x}{\sec^2 x} = \frac{2 \sin x}{\cos x} \cdot \cos^2 x = 2 \sin x \cos x = \text{RHS}$

2. $\tan \theta$ 3. (a) $\frac{1}{2}$ (b) $\frac{\sqrt{2} + \sqrt{6}}{4}$ (c) $\frac{1}{2}\sqrt{2 - \sqrt{3}}$

4. $(10 - 2\sqrt{5})/15$

5. (a) $\frac{1}{2}(\sin 8x - \sin 2x)$ (b) $-2 \cos \frac{7}{2}x \sin \frac{3}{2}x$ 6. -2

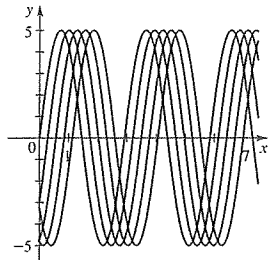
7. (a) 0.34, 2.80 (b) $\pi/3, \pi/2, 5\pi/3$ (c) $2\pi/3, 4\pi/3$

(d) $\pi/6, \pi/2, 5\pi/6, 3\pi/2$ 8. 0.58, 2.56, 3.72, 5.70 9. $\frac{1519}{1681}$

10. $\frac{\sqrt{1-x^2} - xy}{\sqrt{1+y^2}}$

FOCUS ON MODELING ■ PAGE 536

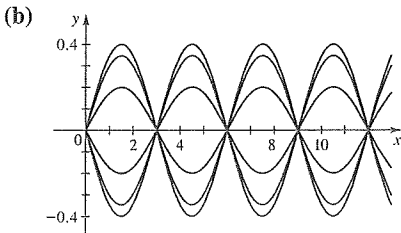
1. (a) $y = -5 \sin\left(\frac{\pi}{2}t\right)$ (b)



(c) $v = \pi/4$

Yes, it is a traveling wave.

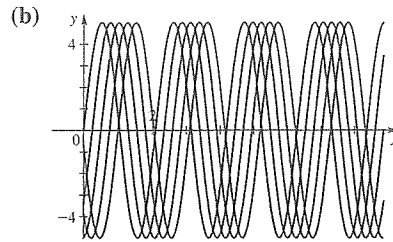
2. (a) $y = 0.4 \sin(1.047x) \cos(0.524t)$; 3, 6, 9, 12, ...



Yes, this is a standing wave.

3. $y(x, t) = 2.7 \sin(0.68x - 4.10t)$

4. (a) $y(x, t) = 5 \sin(3x - 1.5t)$

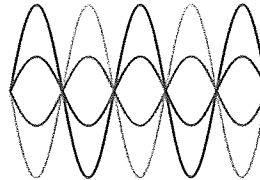


5. $y(x, t) = 0.6 \sin(\pi x) \cos(40\pi t)$

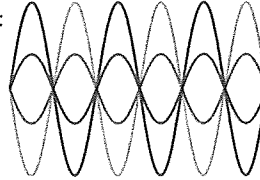
6. $y(x, t) = 7 \sin 2x \cos \frac{1}{2}t$

7. (a) 1, 2, 3, 4

(b) 5:



6:



(c) 880π (d) $y(x, t) = \sin t \cos(880\pi t)$;

$y(x, t) = \sin(2t) \cos(880\pi t)$; $y(x, t) = \sin(3t) \cos(880\pi t)$;

$y(x, t) = \sin(4t) \cos(880\pi t)$

8. (a) $\pi, 3\pi, 5\pi, 7\pi, 9\pi, 11\pi$; no (b) 25 Hz

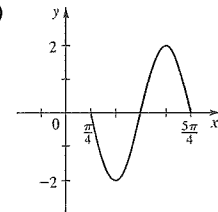
CUMULATIVE REVIEW TEST FOR CHAPTERS 5, 6 AND 7 ■ PAGE 539

1. (a) $\sqrt{5}/3$ (b) $-2/3$ (c) $-\sqrt{5}/2$ (d) $3\sqrt{3}/5$

2. (a) $2\sqrt{10}/7$ (b) $7/3$ (c) $3\sqrt{10}/20$

3. (a) $-\sqrt{3}/2$ (b) -1 (c) $2\sqrt{3}/3$ (d) -1

4. $\sin t = -24/25$, $\tan t = -24/7$, $\cot t = -7/24$, $\sec t = 25/7$,
 $\csc t = -25/24$ 5. (a) $2, \pi, \pi/4$ (b)



6. $y = 3 \cos \frac{1}{2}\left(x - \frac{\pi}{3}\right)$ 7. (a) $h(t) = 45 - 40 \cos 8\pi t$

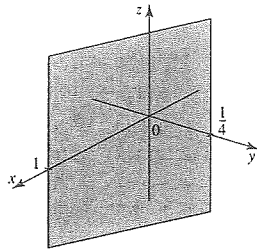
(b) $2\sqrt{19} \approx 8.7$ cm 8. (a) 7.2 (b) 92.9°

9. (a) LHS = $\frac{(\sec \theta - 1)(\sec \theta + 1)}{\tan \theta (\sec \theta + 1)}$

$= \frac{\sec^2 \theta - 1}{\tan \theta (\sec \theta + 1)} = \frac{\tan^2 \theta}{\tan \theta (\sec \theta + 1)} = \text{RHS}$

(b) RHS = $1 - (1 - 2 \sin^2 2\theta) = 2 \sin^2 2\theta = 2(2 \sin \theta \cos \theta)^2$
 $= \text{LHS}$

20. (a) $x + 4y = 1$ (b) x-intercept 1, y-intercept $\frac{1}{4}$, no z-intercept

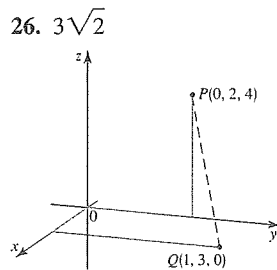
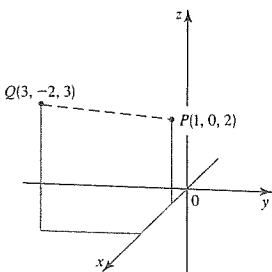


21. $5x - 3y - z = 35$ 22. $x - z = -2$ 23. $x - 3y = 2$
 24. $y + z = 2$ 25. $2x - 3y - 9z = 0$ 26. $2x + 3y + z = 4$
 27. $x = 2t, y = 5t, z = 4 - 4t$
 28. $x = -2 + 2t, y = 0, z = 10t$ 29. $x = 2, y = -1 + t, z = 5$
 30. $x = -3, y = t, z = 2$ 31. $12x + 4y + 3z = 12$
 32. $3x + 6y - 2z = -6$ 33. $x - 2y + 4z = 0$
 34. $12x + 9y - z = 30$

CHAPTER 9 REVIEW ■ PAGE 621

1. $\sqrt{13}, \langle 6, 4 \rangle, \langle -10, 2 \rangle, \langle -4, 6 \rangle, \langle -22, 7 \rangle$
 2. $\sqrt{29}, \langle 2, -2 \rangle, \langle 8, -2 \rangle, \langle 10, -4 \rangle, \langle 21, -6 \rangle$
 3. $\sqrt{5}, 3i - j, i + 3j, 4i + 2j, 4i + 7j$
 4. $3, -i + 5j, i + j, 6j, 2i + 5j$ 5. $\langle 3, -4 \rangle$ 6. $\langle 10, -2 \rangle$
 7. $4, 120^\circ$ 8. $\sqrt{29}, 291.8^\circ$ 9. $\langle 10, 10\sqrt{3} \rangle$ 10. $\langle -7.7, 11.1 \rangle$
 11. (a) $\langle 4.8i + 0.4j \rangle \times 10^4$ (b) 4.8×10^4 lb, N 85.2° E
 12. (a) $\langle 300\sqrt{3} - 25,300 + 25\sqrt{3} \rangle$ (b) 602 mi/h, N 55.2° E
 13. 5, 25, 60 14. 13, 169, 2 15. $2\sqrt{2}, 8, 0$ 16. 10, 100, -30
 17. Yes 18. No, 77.5° 19. No, 45° 20. Yes

21. (a) $\frac{17\sqrt{37}}{37}$ (b) $\langle \frac{102}{37}, -\frac{17}{37} \rangle$
 (c) $\mathbf{u}_1 = \langle \frac{102}{37}, -\frac{17}{37} \rangle, \mathbf{u}_2 = \langle \frac{9}{37}, \frac{54}{37} \rangle$ 22. (a) $-\sqrt{2}$
 (b) $\langle -1, -1 \rangle$ (c) $\mathbf{u}_1 = \langle -1, -1 \rangle, \mathbf{u}_2 = \langle -7, 7 \rangle$
 23. (a) $-\frac{14\sqrt{97}}{97}$ (b) $-\frac{56}{97}\mathbf{i} + \frac{126}{97}\mathbf{j}$
 (c) $\mathbf{u}_1 = -\frac{56}{97}\mathbf{i} + \frac{126}{97}\mathbf{j}, \mathbf{u}_2 = \frac{153}{97}\mathbf{i} + \frac{68}{97}\mathbf{j}$ 24. (a) 4 (b) $4\mathbf{j}$
 (c) $\mathbf{u}_1 = 4\mathbf{j}, \mathbf{u}_2 = 2\mathbf{i}$
 25. 3



27. $x^2 + y^2 + z^2 = 36$
 28. $(x - 1)^2 + (y + 2)^2 + (z - 4)^2 = 4$
 29. Center: $(1, 3, -2)$, radius: 4
 30. Center: $(0, 2, 2)$, radius: $2\sqrt{2}$
 31. $6, \langle 6, 1, 3 \rangle, \langle 2, -5, 5 \rangle, \langle -1, -\frac{15}{2}, 5 \rangle$

32. $10, 7i - j - 7k, 5i + j - 9k, \frac{5}{2}i + 2j - 8k$
 33. (a) -1 (b) No, 92.8° 34. (a) 0 (b) Yes 35. (a) 0
 (b) Yes 36. (a) 1 (b) No, 60° 37. (a) $\langle -2, 17, -5 \rangle$

(b) $\langle -\frac{\sqrt{318}}{159}, \frac{17\sqrt{318}}{318}, -\frac{5\sqrt{318}}{318} \rangle$ 38. (a) $\langle -3, 2, 8 \rangle$

(b) $\langle -\frac{3\sqrt{77}}{77}, \frac{2\sqrt{77}}{77}, \frac{8\sqrt{77}}{77} \rangle$ 39. (a) $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$

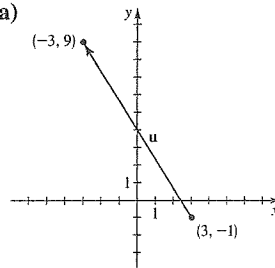
(b) $\frac{\sqrt{6}}{6}\mathbf{i} + \frac{\sqrt{6}}{6}\mathbf{j} + \frac{\sqrt{6}}{3}\mathbf{k}$ 40. (a) $-2\mathbf{j} - 2\mathbf{k}$

(b) $-\frac{\sqrt{2}}{2}\mathbf{j} - \frac{\sqrt{2}}{2}\mathbf{k}$ 41. $\frac{15}{2}$ 42. $9\sqrt{2}$ 43. 9 44. 14

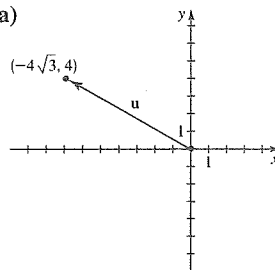
45. $x = 2 + 3t, y = t, z = -6$
 46. $x = 5 + 2t, y = 2 - t, z = 8 + 5t$
 47. $x = 6 - 2t, y = -2 + 3t, z = -3 + t$
 48. $x = 1 + 2t, y = -4t, z = 2t$
 49. $2x + 3y - 5z = 2$ 50. $x + 2y - 7z = -6$
 51. $7x + 7y + 6z = 20$ 52. $15x - 20y - 12z = 60$
 53. $x = 2 - 2t, y = 0, z = -4t$ 54. $4x - 2y - z = 14$

CHAPTER 9 TEST ■ PAGE 623

1. (a) (b) $-6\mathbf{i} + 10\mathbf{j}$ (c) $2\sqrt{34}$



2. (a) $\langle 19, -3 \rangle$ (b) $5\sqrt{2}$ (c) 0 (d) Yes
 3. (a) (b) 8, 150°



4. (a) $14\mathbf{i} + 6\sqrt{3}\mathbf{j}$ (b) 17.4 mi/h, N 53.4° E 5. (a) 45.0°
 (b) $\frac{\sqrt{26}}{2}$ (c) $\frac{5}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}$ 6. 90 7. (a) 6
 (b) $(x - 4)^2 + (y - 3)^2 + (z + 1)^2 = 36$
 (c) $\langle 2, -4, 4 \rangle = 2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$ 8. (a) $11\mathbf{i} - 4\mathbf{j} - \mathbf{k}$ (b) $\sqrt{6}$
 (c) -1 (d) $-3\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}$ (e) $3\sqrt{35}$ (f) 18 (g) 96.3°

9. $\langle \frac{7\sqrt{6}}{18}, \frac{\sqrt{6}}{9}, -\frac{\sqrt{6}}{18} \rangle, \langle -\frac{7\sqrt{6}}{18}, -\frac{\sqrt{6}}{9}, \frac{\sqrt{6}}{18} \rangle$

10. (a) $\langle 4, -3, 4 \rangle$ (b) $4x - 3y + 4z = 4$ (c) $\frac{\sqrt{41}}{2}$

11. $x = 2 - 2t, y = -4 + t, z = 7 - 2t$