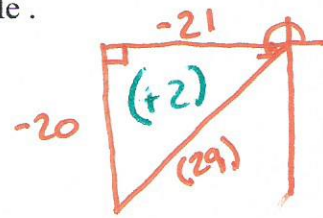


3. (16 points) Use double-angle or half-angle formulas to evaluate the following expressions.

a. $\cos 2x$, if $\tan x = \frac{20}{21}$ and x is a quadrant III angle.

$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x && (+2) \\ &= 1 - 2\sin^2 x && \text{(either)} \\ &= 2\cos^2 x - 1 \end{aligned}$$



$$\begin{aligned} (-20)^2 + (-21)^2 &= c^2 \\ 400 + 441 &= c^2 \\ 841 &= c^2 \\ 841 &= 29^2 \end{aligned}$$

$$\left(\frac{-21}{29}\right)^2 - \left(\frac{-20}{29}\right)^2 = \frac{441 - 400}{841} = \frac{41}{841} \quad (+2)$$

b. $\sin 105^\circ$

$$\begin{aligned} \sin 105^\circ &= \sin\left(\frac{210^\circ}{2}\right) = \pm \sqrt{\frac{1 - \cos 210^\circ}{2}} && (+2) \\ &= \pm \sqrt{\frac{1 - (-\sqrt{3}/2)}{2}} = \sqrt{\frac{1 + \sqrt{3}/2}{2}} \cdot \frac{2}{2} && (+2) \\ &= \frac{1}{2} \sqrt{2 + \sqrt{3}} && (+2) \end{aligned}$$

\downarrow
 $\frac{2 + \sqrt{3}}{4}$



⊕ since 105° is in II
 $\frac{S}{A} = \frac{C}{R}$

4. (16 points) Find all the solutions to each equation.

a. $2\cos^2 x + \cos x - 1 = 0$

think $2x^2 + x - 1 = 0$
 $(2x - 1)(x + 1) = 0$

$$(2\cos x - 1)(\cos x + 1) = 0 \quad (+2)$$

$$2\cos x - 1 = 0$$

$$\cos x + 1 = 0$$

$$\cos x = \frac{1}{2}$$

$$\cos x = -1 \quad (+2)$$

$$x = \frac{\pi}{3} + 2k\pi \quad (+2)$$

$$x = \pi + 2k\pi \quad (+2)$$

$$x = -\frac{\pi}{3} + 2k\pi$$

b. $3\tan^2 x - 1 = 0$, $0 \leq x < 2\pi$ ← positive angles 0-360°

$$3\tan^2 x = 1 \quad (+2)$$

$$\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm \sqrt{\frac{1}{3}} = \pm \frac{\sqrt{3}}{3} \quad (+2)$$

