

Eastern Oregon University
Concurrent Enrollment/Credit by Proficiency Program

Math 112, Spring, 2016

Exam 3

name/school: Key

Show any relevant work. For each problem, circle your answer.

1. (20 points) Verify each of the following identities:

10 a. $(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x$

$$\begin{aligned} (\sin x + \cos x)^2 &= \sin^2 x + 2 \sin x \cos x + \cos^2 x \quad] + 6 \\ &= \sin^2 x + \cos^2 x + 2 \sin x \cos x \quad] + 2 \\ &= 1 + 2 \sin x \cos x \quad] + 2 \end{aligned}$$

10 b. Use a sum or difference formula to verify: $\sin(x+y) - \sin(x-y) = 2 \cos x \sin y$

$$\begin{aligned} \sin(x+y) - \sin(x-y) &= \sin x \cos y + \cos x \sin y - (\sin x \cos y - \cos x \sin y) \\ &= \sin x \cos y + \cos x \sin y - \sin x \cos y + \cos x \sin y \\ &= 2 \cos x \sin y \end{aligned}$$

2. (16 points) Find all solutions to each equation in the interval $0 \leq \theta \leq 2\pi$:

8 a. $\cos^2 \theta (2 \cos \theta - 1) = 0$

$$\begin{aligned} \cos^2 \theta = 0 &\quad \text{or} \quad 2 \cos \theta - 1 = 0 \\ \cos \theta = 0 &\quad \text{or} \quad \cos \theta = \frac{1}{2} \\ \theta = \frac{\pi}{2} \text{ or } \frac{3\pi}{2} &\quad \text{or} \quad \theta = \frac{\pi}{3} \text{ or } \frac{5\pi}{3} \end{aligned}$$

8 b. $\cos 2\theta - \cos^2 \theta = 0$

$$\begin{aligned} \cos 2\theta - \cos^2 \theta &= (\cos^2 \theta - \sin^2 \theta) - \cos^2 \theta \\ &= -\sin^2 \theta = 0 \quad \text{iff} \quad \sin \theta = 0 \\ &\quad \text{iff} \quad \theta = 0 \text{ or } \pi \end{aligned}$$

3. (24 points) Use addition or subtraction formulas, double-angle or half-angle formulas as appropriate to evaluate each of the following expressions.

6 a. $\sin \frac{\pi}{12}$

with $\theta = \frac{\pi}{6}$, then $\frac{\pi}{12} = \frac{\theta}{2}$] + 2

+4 $\left[\sin \frac{\pi}{12} = \frac{+}{-} \sqrt{\frac{1 - \cos \frac{\pi}{6}}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} \right]$
 $= \sqrt{\frac{2 - \sqrt{3}}{4}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$

or $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$, so
 $\sin \frac{\pi}{12} = \sin \left(\frac{\pi}{3} - \frac{\pi}{4} \right)$] + 2
 $= \sin \frac{\pi}{3} \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \sin \frac{\pi}{4}$
 $= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$] + 4
 $= \frac{\sqrt{6} - \sqrt{2}}{4}$

Suppose $\cos x = \frac{2}{5}$ and x is a quadrant IV angle. Find each of the following:

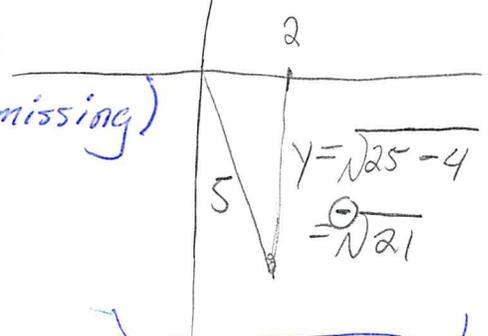
6 b. $\cos 2x$

$\cos 2x = \cos^2 x - \sin^2 x$] - 2
 $= \left(\frac{2}{5} \right)^2 - \left(-\frac{\sqrt{21}}{5} \right)^2$] + 2
 $= \frac{4}{25} - \frac{21}{25} = -\frac{17}{25}$] + 2

simplification not required, but must be correct if attempted.

6 c. $2 \sin 2x$

$= 2 \cdot 2 \sin x \cdot \cos x$] + 2
 $= 4 \left(-\frac{\sqrt{21}}{5} \right) \left(\frac{2}{5} \right) = -\frac{8\sqrt{21}}{25}$] + 2



6 d. $\cos \frac{x}{2}$ = $\frac{+}{-} \sqrt{\frac{1 + \frac{2}{5}}{2}}$] + 4

$= \frac{+}{-} \sqrt{\frac{7}{5}} = \frac{+}{-} \sqrt{\frac{7}{10}}$] + 2

either alone is also O.K.

4. (24 points) Let $\mathbf{u} = \langle 3, 5 \rangle$, $\mathbf{v} = \langle -1, 4 \rangle$. Find each of the following:

6 a. $2\mathbf{u} - \mathbf{v}$

$$= 2\langle 3, 5 \rangle - \langle -1, 4 \rangle \quad]+4$$
$$= \langle 6, 10 \rangle - \langle -1, 4 \rangle = \langle 7, 6 \rangle \quad]+2$$

6 b. $\mathbf{u} \cdot \mathbf{v}$

$$= \langle 3, 5 \rangle \cdot \langle -1, 4 \rangle = 3(-1) + 5 \cdot 4 \quad]+4$$
$$= -3 + 20$$
$$= 17 \quad]+2$$

6 c. $\text{proj}_{\mathbf{v}} \mathbf{u}$

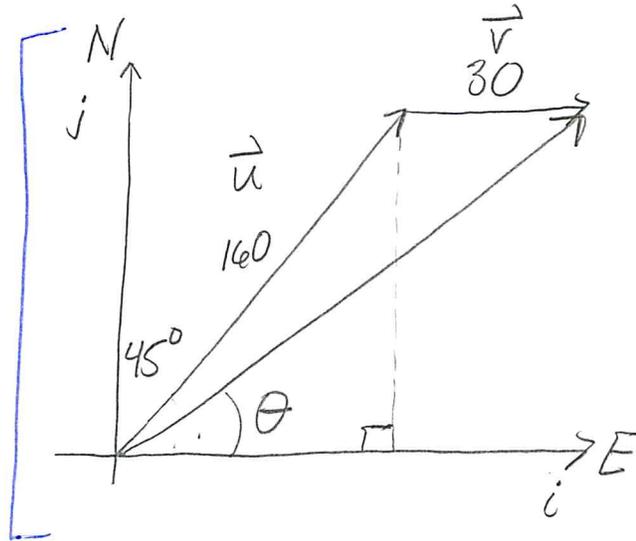
$$= \left(\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2} \right) \mathbf{v} = \frac{17}{17} \mathbf{v} = \mathbf{v} = \langle -1, 4 \rangle \quad]+2$$
$$|\mathbf{v}|^2 = (-1)^2 + 4^2 = 17 \quad]+2 \quad]+2$$

6 d. Resolve \mathbf{u} into \mathbf{u}_1 and \mathbf{u}_2 such that \mathbf{u}_1 is parallel to \mathbf{v} and \mathbf{u}_2 is perpendicular to \mathbf{v} .

$$\mathbf{u}_1 = \text{proj}_{\mathbf{v}} \mathbf{u} = \mathbf{v} = \langle -1, 4 \rangle \quad]+2$$
$$\mathbf{u}_2 = \mathbf{u} - \text{proj}_{\mathbf{v}} \mathbf{u} = \mathbf{u} - \mathbf{v} \quad]+2$$
$$= \langle 3, 5 \rangle - \langle -1, 4 \rangle$$
$$= \langle 4, 1 \rangle \quad]+2$$

5. (16 points) A small plane is flying through a wind which is blowing 30 mph in direction due east. The plane has a speed of 160 mph relative to air and is headed in the direction of N 45° E. Find the true speed and direction of the jet.

not required,
but up to 8
points if
complete



$$\vec{u} = 160 \cdot \cos 45^\circ \vec{i} + 160 \sin 45^\circ \vec{j} \quad]+3$$

$$\vec{v} = 30\vec{i} + 0\vec{j} \quad]+3$$

True speed and direction are given by $|\vec{u} + \vec{v}|$ and θ

$$\vec{u} + \vec{v} = \left(160 \cdot \frac{\sqrt{2}}{2} + 30\right)\vec{i} + 160\left(\frac{\sqrt{2}}{2}\right)\vec{j} \quad]+2$$

$$= (80\sqrt{2} + 30)\vec{i} + 80\sqrt{2}\vec{j}$$

$$|\vec{u} + \vec{v}| = \sqrt{(80\sqrt{2} + 30)^2 + (80\sqrt{2})^2} \approx 182.45 \text{ mp.h.} \quad]+3$$

$$\tan \theta = \frac{80\sqrt{2}}{80\sqrt{2} + 30} \Rightarrow \theta \approx 38.32^\circ \quad]+3$$

This can be stated as E 38.32° N or N 51.68° E]+2

could also have projected \vec{u} onto \vec{j} -axis