Up and Down or Down and Up

11.1

Exploring Quadratic Functions

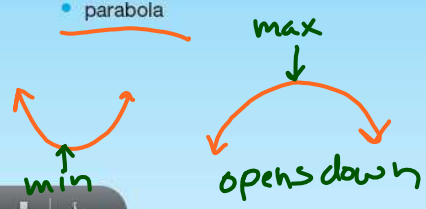
LEARNING GOALS

In this lesson, you will:

- Model real-world problems using quadratic functions.
- Analyze tables, graphs, and equations for quadratic functions.
- Use the Distributive Property to write a quadratic equation in standard form.
- Compare graphs of quadratic functions.
- Use a graphing calculator to determine the absolute minimum or absolute maximum of a quadratic function.

KEY TERMS

- standard form (general form) of a quadratic function
- parabola



opens up
$a>0$
positive

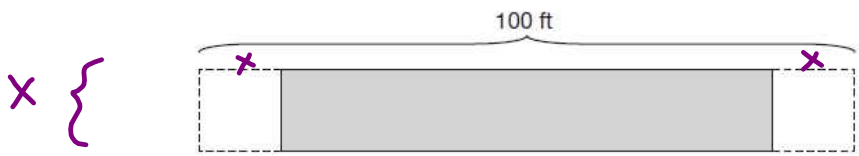
$a<0$
negative

Standard Form for a Quadratic:

$f(x) = ax^2+bx+c$

PROBLEM 1 Who Put the Dogs In?

A dog trainer is fencing in an enclosure, represented by the shaded region in the diagram. The trainer will also have two square-shaped storage units on either side of the enclosure to store equipment and other materials. She can make the enclosure and storage units as wide as she wants, but she can't exceed 100 feet in total length.



1. Let s represent a side length, in feet, of one of the square storage units.
   a. Write an expression to represent the width of the enclosure. Label the width in the diagram. Explain your reasoning.

x

b. Write an expression to represent the length of the enclosure. Label the length in the diagram. Explain your reasoning.

$100 - 2x$

$A = l \cdot w$

c. Write an expression to represent the area of the enclosure. Explain your reasoning.

$y = x(100 - 2x)$

You know that you can use the Distributive Property to rewrite a mathematical expression.

For example, to rewrite $2(5 - 4)$, you can distribute the 2:

$$2(5 - 4) = 2(5) - 2(4)$$

You can also distribute variables:

$$n(4 - 3n) = n(4) - n(3n)$$
$$= 4n - 3n^2$$

The expression $n \cdot 3n$ is equivalent to $n \cdot 3 \cdot n$, which is the same as $n \cdot n \cdot 3$, or $n^2 \cdot 3$, or $3n^2$.

2. Use the Distributive Property to rewrite the expression representing the area of the enclosure. Show your work.

$x(100-2x) = 100x - 2x^2$

The expression you wrote is a quadratic expression. Recall that a quadratic function written in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$, is in **standard form**, or **general form**. In this form, a and b represent numerical coefficients and c represents a constant.

3. Write the area of the enclosure as a function, $A(s)$, in standard form. Then, identify a, b, and c for the function.

$a = -2$
$b = 100$
$c = 0$ goes in

$f(x) = -2x^2 + 100x$

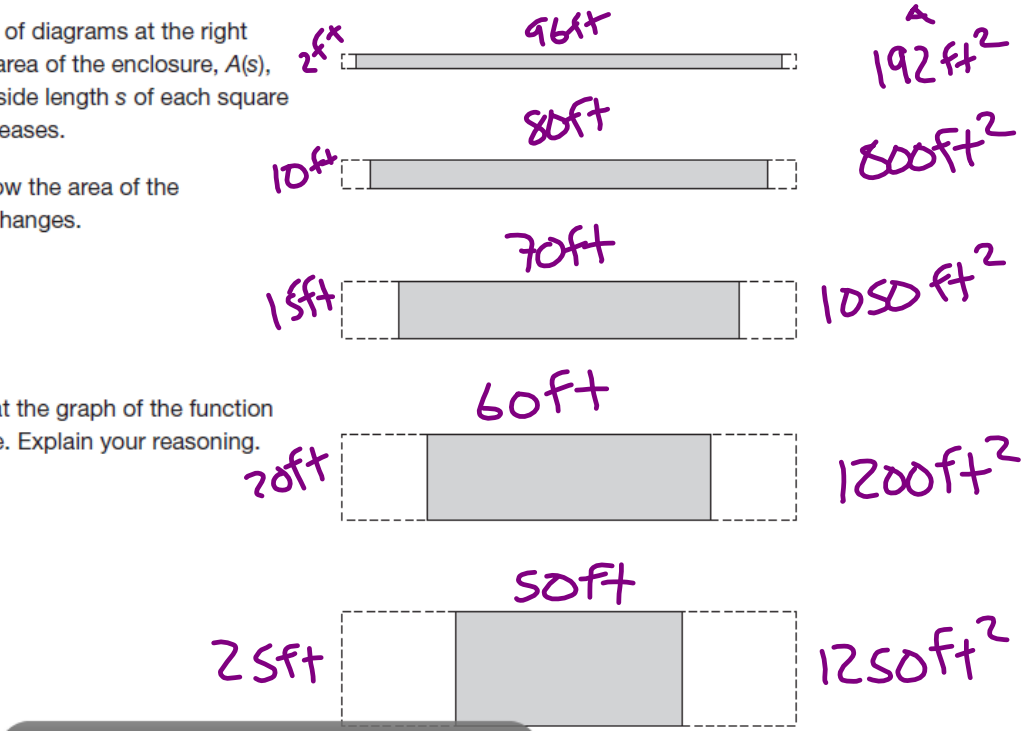
4. Identify the independent and dependent quantities and their units in this problem situation.

inde = width, x
dep = Area of enclosure

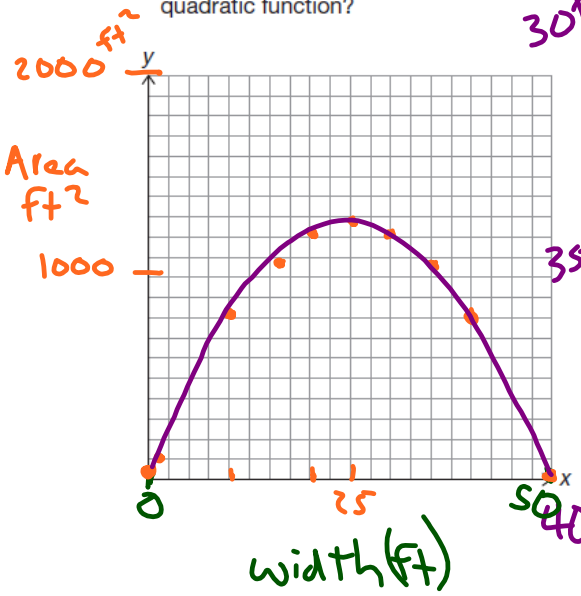
The progression of diagrams at the right shows how the area of the enclosure, $A(s)$, changes as the side length s of each square storage unit increases.

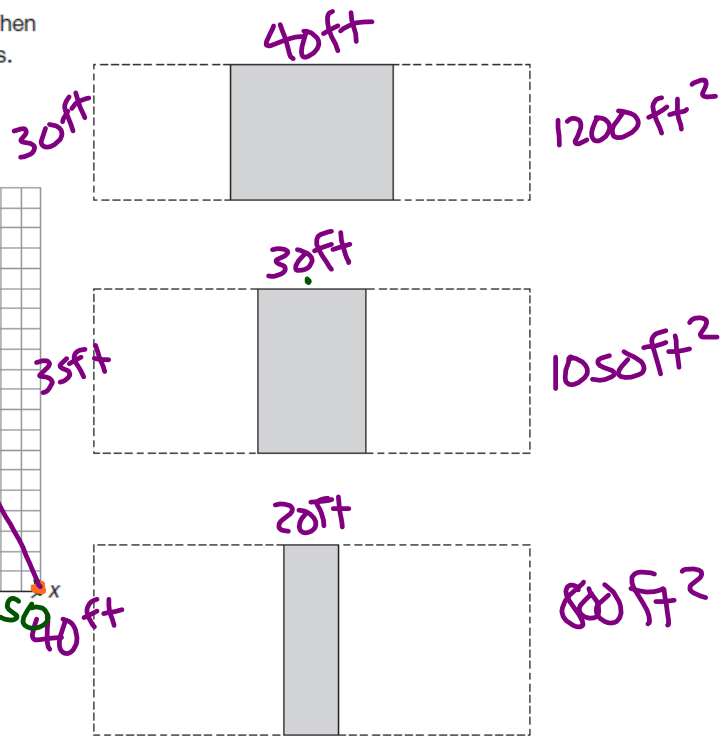
5. Describe how the area of the enclosure changes.

6. Predict what the graph of the function will look like. Explain your reasoning.



7. Use a graphing calculator to graph the function you wrote in Question 3. Then sketch the graph and label the axes. How can you tell the graph is a quadratic function?





$f(x) = -2x^2 + 100x$

$a = -2$

25 ft x 50 ft max area