# 9.2 Notes: The Dot Product

<u>Dot Product</u> is one way vectors are multiplied.
(the other is the <u>cross product</u>)

$u = \langle a_1, b_1 \rangle$

$v = \langle a_2, b_2 \rangle$

$$\underset{\substack{\uparrow \\ \text{dot}}}{u \cdot v} = a_1 \cdot a_2 + b_1 \cdot b_2 \leftarrow$$ The Dot product is a Scalar

ex: $w = \langle 5, 1 \rangle$

$y = \langle 6, -3 \rangle$

$w \cdot y = 5 \cdot 6 + 1 \cdot -3 = 30 - 3 = 27$

$m = \langle -2, 4 \rangle$

$m \cdot n = -2 \cdot 1 + 4 \cdot -4 = -2 - 16 = -18$

$n = \langle 1, -4 \rangle$

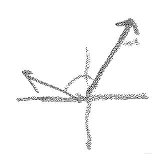
## The Angle Between Two Vectors

$$\cos\theta = \frac{u \cdot v}{|u||v|} \quad \text{or} \quad \cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right) = \theta$$

$\uparrow$ the angle between $u$ & $v$

ex: find the Angle between

$w = \langle 3, 4 \rangle$ and $y = \langle -2, 1 \rangle$



$$\cos\theta = \frac{3 \cdot -2 + 4 \cdot 1}{\sqrt{9+16} \cdot \sqrt{4+1}} = \frac{-6+4}{5 \cdot \sqrt{5}} = \frac{-2}{5\sqrt{5}} \qquad \cos^{-1}\left(\frac{-2}{5\sqrt{5}}\right) = 100.3^\circ$$

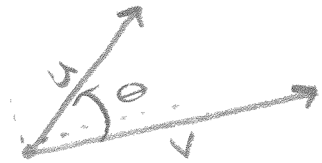
- Orthogonal (perpendicular) vectors have a dot product equal to zero.

$$u \cdot v = 0 \quad \Rightarrow \theta = \frac{\pi}{2} \text{ and } \cos\frac{\pi}{2} = 0$$

$\uparrow$ you can use this to check vectors for perpedicularity

## The Component of u along v

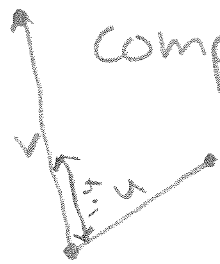
What part of vector u goes in the same direction as vector v




u is broken up into perpendicular components 1 of them in the same direction as v

$$\text{comp}_v(u) = |u|\cos\theta = \frac{u \cdot v}{|v|}$$

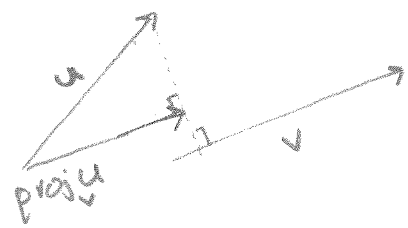
ex: $u = \langle 3, 2 \rangle$
$v = \langle -1, 5 \rangle$




$$\text{comp}_v(u) = \frac{3 \cdot -1 + 2 \cdot 5}{\sqrt{(-1)^2 + (5)^2}} = \frac{-3+10}{\sqrt{26}} = \frac{7}{\sqrt{26}}$$

$$= 1.4$$

## The Projection of u onto v




$\text{proj}_v u$ is the vector parallel to v whose length is $\text{comp}_v u$

$$\text{proj}_v u = \left(\frac{u \cdot v}{|v|}\right) \cdot \frac{v}{|v|} = \left(\frac{u \cdot v}{|v|^2}\right) v$$

unit vector in the direction of v

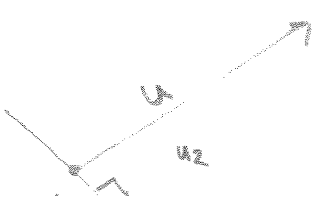
we write $u = u_1 + u_2$ where $u_1 \parallel v$ and $u_2 \perp v$ (orthoganal)

$$u_1 = \text{proj}_v u \qquad u_2 = u - \text{proj}_v u$$

ex: $u = \langle 5, 3 \rangle$
$v = \langle -2, 1 \rangle$

$$\text{proj}_v u = \left(\frac{5 \cdot -2 + 3 \cdot 1}{\sqrt{(-2)^2 + (1)^2}^{\,2}}\right) \cdot \langle -2, 1 \rangle = \left(\frac{\overset{-7}{-10+3}}{5}\right) \langle -2, 1 \rangle$$

$$= \left\langle \frac{14}{5}, \frac{-7}{5} \right\rangle$$





$$u_1 = \left\langle \frac{14}{5}, \frac{-7}{5} \right\rangle$$

$$u_2 = u - u_1 = \langle 5,3 \rangle - \langle \frac{14}{5}, \frac{-7}{5} \rangle = \langle \frac{11}{5}, \frac{22}{5} \rangle$$

<u>Work</u>

The work done by force F in moving along vector D

$$W = F \cdot D$$

$F = \langle 4,3 \rangle$ & moves an object from $\langle 1,5 \rangle$ to $\langle 4,8 \rangle$

$$D = \langle 4-1, 8-5 \rangle = \langle 3,3 \rangle$$

$$W = \langle 4,3 \rangle \cdot \langle 3,3 \rangle = 12 + 9 = 21 \text{ ft pounds}$$