

Section 8.3: p. 562; # 1-4, 5-15 odd, 17-19, 21, 23-25, 29-32, 37-43, 45, 47, 53, 55, 57, 61, 63, 69, 70, 72, 75, 81-85, 87, 91, 92, 94

1) A complex number has a real part  $a$  & an imaginary part  $b$ . To graph  $a+bi$ , we graph the point  $(a,b)$  on the complex plane.

2) a)  $z = a+bi$ : has modulus  $|z| = r = \sqrt{a^2+b^2}$ .

An argument of  $z$  is angle  $\theta$ , such that  $\tan \theta = \frac{b}{a}$ .

b)  $z$  in polar form =  $r(\cos \theta + i \sin \theta)$   
 ↑ modulus      ↑ argument

3) a)  $z = -1 + i \rightarrow \sqrt{2}(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4})$  in polar form

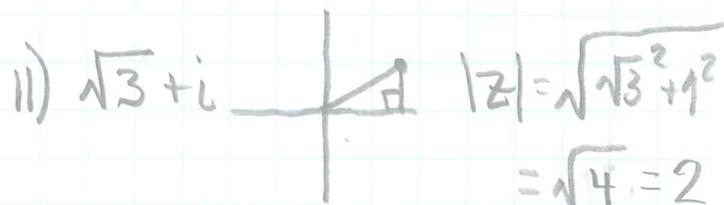
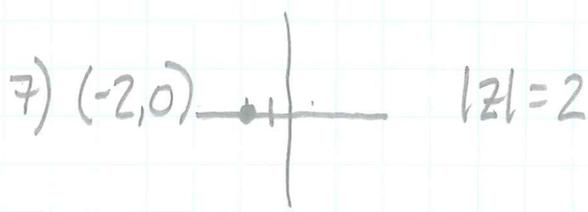
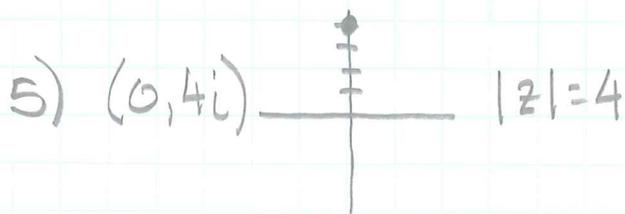
$z = 2(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}) \rightarrow (\sqrt{3}, 1)$  in rectangular form

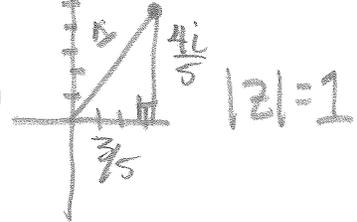
$$2 \cos \frac{\pi}{6} = 2 \cdot \frac{\sqrt{3}}{2} \quad 2 \sin \frac{\pi}{6} = 2 \cdot \frac{1}{2} \quad \text{or } \sqrt{3} + i$$

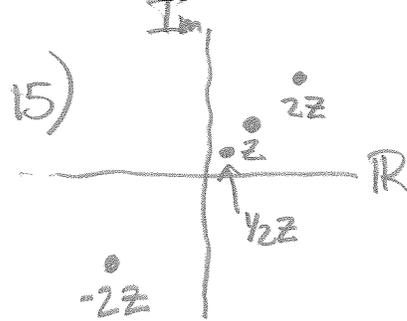
b)  $1+i$ : rectangular.  $(\sqrt{2}, \frac{\pi}{4}) = z = \sqrt{2}(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$

4) A nonzero complex Number has  $n$ ,  $n^{\text{th}}$  roots

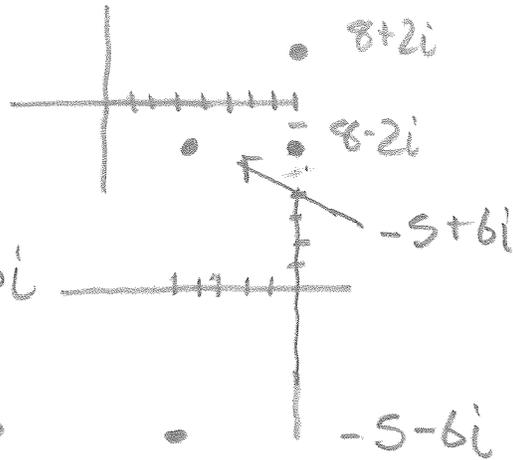
The number 16 has 4,  $4^{\text{th}}$  roots;  $\pm 2, \pm 2i$



13)  $\left(\frac{3}{5} + \frac{4i}{5}\right)$    $|z|=1$

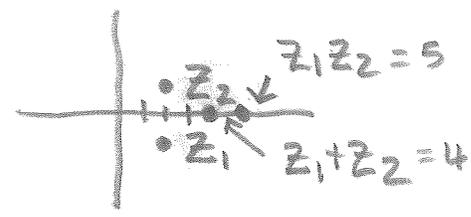


17)  $8+2i$  has conjugate  $8-2i$



18)  $-5+6i$  has conjugate  $-5-6i$

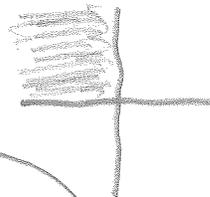
19)  $z_1 = 2-i$   
 $z_2 = 2+i$



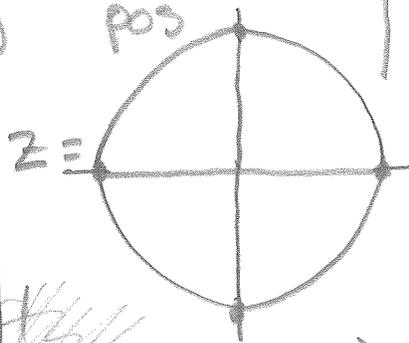
$$(2-i)(2+i)$$

$$4 + 2i - 2i - (-1) = 4 + 1 = 5$$

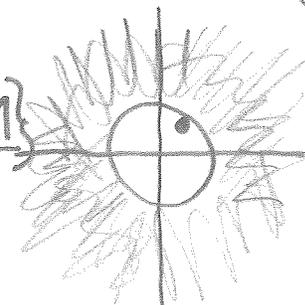
21)  $\{z = a+bi \mid a \leq 0, b \geq 0\}$   
 neg pos



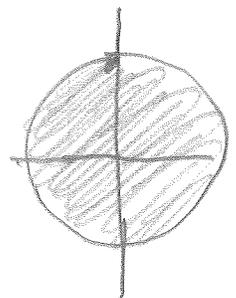
23)  $\{z \mid |z| = 3\}$



24)  $\{z \mid |z| \geq 1\}$



25)  $\{z \mid |z| < 2\}$



29)  $|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$$z = \sqrt{2} \sqrt{\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}}$$

$$\tan^{-1} 1 = \frac{\pi}{4}$$

30)  $|z| = \sqrt{1^2 + \sqrt{3}^2} = \sqrt{4} = 2$

$$\tan^{-1} \frac{\sqrt{3}}{1} = \frac{\pi}{3}$$

$$z = 2 \sqrt{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$$

$$31) |z| = \sqrt{(\sqrt{2})^2 + (-\sqrt{2})^2} = \sqrt{4} = 2$$

$$\tan^{-1}\left(\frac{\sqrt{2}}{-\sqrt{2}}\right) = \tan^{-1}(-1) = \frac{7\pi}{4}$$

$$z = 2\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

$$32) |z| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$\tan^{-1}\left(\frac{-1}{1}\right) = \tan^{-1}(-1) = \frac{7\pi}{4}$$

$$z = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

$$37) |z| = \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$$

$$\tan^{-1}\left(\frac{5}{5}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

$$z = 5\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$38) |z| = \sqrt{4^2 + 0^2} = \sqrt{16} = 4$$

$$\tan^{-1}\left(\frac{0}{4}\right) = \tan^{-1}(0) = 0$$

$$z = 4(\cos 0 + i\sin 0)$$

$$39) |z| = \sqrt{(4\sqrt{3})^2 + (-4)^2} = \sqrt{48+16} = \sqrt{64} = 8$$

$$\tan^{-1}\left(\frac{-4}{4\sqrt{3}}\right) = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{-\pi}{6} = \frac{11\pi}{6}$$

$$z = 8\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$$

$$40) |z| = \sqrt{0^2 + 8^2} = 8$$

$$\tan^{-1}\left(\frac{8}{0}\right) = \tan^{-1}(\text{undef}) = \frac{\pi}{2}$$

$$z = 8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$41) |z| = \sqrt{(-20)^2 + 0^2} = 20$$

$$\tan^{-1}\left(\frac{0}{-20}\right) = \tan^{-1}(0) = \pi$$

$$z = 20(\cos\pi + i\sin\pi)$$

$$42) |z| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$$

$$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

$$z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$$43) |z| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \quad z = 5(\cos(\tan^{-1}(4/3)) + i\sin(\tan^{-1}(4/3)))$$

$$\tan^{-1}(4/3) = \theta$$

$$45) 3i(1+i) = 3i + 3i^2$$

$$i^2 = -1 \text{ so } = -3 + 3i \quad |z| = \sqrt{(-3)^2 + (3)^2} = \sqrt{18} = 3\sqrt{2}$$

$$\tan^{-1}\left(\frac{3}{-3}\right) = \tan^{-1}(-1) = \frac{3\pi}{4} \quad z = 3\sqrt{2} \left( \cos \frac{3\pi}{4} + i\sin \frac{3\pi}{4} \right)$$

$$47) 4(\sqrt{3} + i) = 4\sqrt{3} + 4i$$

$$|z| = \sqrt{(4\sqrt{3})^2 + 4^2} = \sqrt{48 + 16} = \sqrt{64} = 8$$

$$\tan^{-1}\left(\frac{4}{4\sqrt{3}}\right) = \frac{\pi}{6} \quad z = 8 \left( \cos \frac{\pi}{6} + i\sin \frac{\pi}{6} \right)$$

$$53) z_1 = \cos \pi + i \sin \pi \quad z_2 = \cos \frac{\pi}{3} + i \sin \frac{\pi}{3}$$

$$z_1 z_2 = \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}$$

$$z_1 / z_2 = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$$

$$\pi + \frac{\pi}{3} = \frac{3\pi}{3} + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$\pi - \frac{\pi}{3} = \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$55) z_1 = 3(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}) \quad z_2 = 5(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3})$$

$$z_1 z_2 = 15(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2})$$

$$z_1 / z_2 = \frac{3}{5}(\cos -\frac{7\pi}{6} + i \sin -\frac{7\pi}{6})$$

$$\frac{\pi}{6} + \frac{4\pi}{3} = \frac{\pi}{6} + \frac{8\pi}{6} = \frac{9\pi}{6} = \frac{3\pi}{2}$$

$$\frac{\pi}{6} - \frac{4\pi}{3} = \frac{\pi}{6} - \frac{8\pi}{6} = -\frac{7\pi}{6}$$

$$57) z_1 = 4(\cos 120^\circ + i \sin 120^\circ) \quad z_2 = 2(\cos 30^\circ + i \sin 30^\circ)$$

$$z_1 z_2 = 8(\cos 150^\circ + i \sin 150^\circ)$$

$$z_1 / z_2 = 2(\cos 90^\circ + i \sin 90^\circ)$$

$$61) z_1 = \sqrt{3} + i = 2(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}) \quad z_2 = 1 + \sqrt{3}i = 2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$$

$$r = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$$

$$\tan \theta = \frac{1}{\sqrt{3}} \quad \theta = \frac{\pi}{6}$$

$$r = 2$$

$$\tan \theta = \frac{\sqrt{3}}{1} = \sqrt{3} \quad \theta = \frac{\pi}{3}$$

$$z_1 z_2 = 4(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}) \quad z_1 / z_2 = \cos -\frac{\pi}{6} + i \sin -\frac{\pi}{6}$$

$$= \cos \frac{\pi}{6} - i \sin \frac{\pi}{6}$$

$$63) z_1 = 2\sqrt{3} - 2i = 4(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}) \quad z_2 = -1 + i = \sqrt{2}(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4})$$

$$r = \sqrt{(2\sqrt{3})^2 + (2)^2} = \sqrt{16} = 4$$

$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\tan \theta = \frac{-2}{2\sqrt{3}} = -\frac{1}{\sqrt{3}} \quad \theta = -\frac{\pi}{6} = \frac{11\pi}{6}$$

$$\tan \theta = 1 \quad \tan^{-1} 1 = \theta = \frac{\pi}{4} \text{ or } \frac{3\pi}{4}$$

$$z_1 z_2 = 4\sqrt{2}(\cos \frac{7\pi}{12} + i \sin \frac{7\pi}{12}) \quad z_1 / z_2 = 2\sqrt{2}(\cos \frac{13\pi}{4} + i \sin \frac{13\pi}{4})$$

$$\frac{4\sqrt{2}}{2\sqrt{2}} \rightarrow$$

$$69) (1+i)^{20} = \sqrt{2}^{20} \left( \cos \frac{20\pi}{4} + i \sin \frac{20\pi}{4} \right) = 2^{10} (\cos 5\pi + i \sin 5\pi)$$

$$\tan \theta = 1/1 \quad \tan^{-1} 1 = \pi/4$$

$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

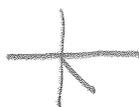
$$= 1024 (\cos \pi + i \sin \pi)$$

$$= 1024 (-1 + 0) = -1024$$

$$70) (1 - \sqrt{3}i)^5 = 2^5 \left( \cos \frac{(5)5\pi}{3} + i \sin \frac{(5)5\pi}{3} \right)$$

$$r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$$

$$\tan \theta = -\frac{\sqrt{3}}{1} \quad \tan^{-1} -\sqrt{3} = -\frac{\pi}{3} = \frac{5\pi}{3}$$



$$= 32 \left( \cos \frac{25\pi}{3} + i \sin \frac{25\pi}{3} \right)$$

$$= 32 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$= 32 \left( \frac{1}{2} + i \frac{\sqrt{3}}{2} \right) = 16 + 16\sqrt{3}i$$

$$72) (1-i)^8 = \sqrt{2}^8 = 2^4 \left( \cos \frac{2(8)7\pi}{4} + i \sin \frac{(8)7\pi}{4} \right)$$

$$r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$\tan \theta = -1/1, \quad \tan^{-1}(-1) = -\frac{\pi}{4} = \frac{7\pi}{4}$$



$$= 2^4 \left( \cos 14\pi + i \sin 14\pi \right)$$

$$= 16 (\cos 0 + i \sin 0)$$

$$= 16 (1 + 0) = 16$$

$$75) (2-2i)^8 = (2\sqrt{2})^8 \left( \cos \frac{2(8)7\pi}{4} + i \sin \frac{2(8)7\pi}{4} \right)$$

$$r = \sqrt{2^2 + (-2)^2} = \sqrt{4+4} = 2\sqrt{2}$$

$$\theta = -\frac{\pi}{4} = \frac{7\pi}{4}$$

$$= 4096$$

$$\frac{\theta}{2} + k\pi$$

$$81) (4\sqrt{3} + 4i)^{1/2}$$

$$r = \sqrt{(4\sqrt{3})^2 + 4^2} = \sqrt{64} = 8$$

$$\tan \theta = \frac{4}{4\sqrt{3}} = \frac{1}{\sqrt{3}}; \quad \theta = \frac{\pi}{6}$$

$$= 8^{1/2} \left( \cos \frac{\theta + 2k\pi}{2} + i \sin \frac{\theta + 2k\pi}{2} \right)$$

$$2\sqrt{2} \left( \cos \frac{\pi/6}{2} + i \sin \frac{\pi/6}{2} \right)$$

$$2\sqrt{2} \left( \cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right)$$

$$\& 2\sqrt{2} \left( \cos \frac{13\pi}{12} + i \sin \frac{13\pi}{12} \right)$$

$$82) \sqrt[3]{8 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)} = 8^{\frac{1}{3}} \left( \cos \frac{\theta + 2k\pi}{3} + i \sin \frac{\theta + 2k\pi}{3} \right)$$

$$k=0,1,2$$

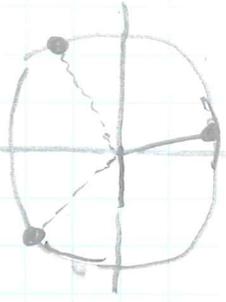
$$\frac{2k\pi}{3} = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\text{or } \frac{12\pi}{18}, \frac{24\pi}{18}$$

$$\bullet 2 \left( \cos \frac{\pi}{18} + i \sin \frac{\pi}{18} \right)$$

$$\bullet 2 \left( \cos \frac{13\pi}{18} + i \sin \frac{13\pi}{18} \right)$$

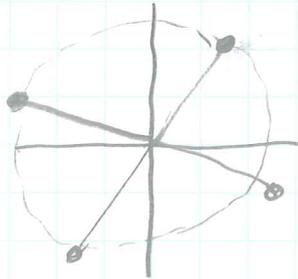
$$\bullet 2 \left( \cos \frac{25\pi}{18} + i \sin \frac{25\pi}{18} \right)$$



$$83) \sqrt[4]{-81i} = \sqrt[4]{81 \left( \cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right)} = 81^{\frac{1}{4}} \left( \cos \frac{\theta + 2k\pi}{4} + i \sin \frac{\theta + 2k\pi}{4} \right)$$

$$r = \sqrt{0^2 + (-81)^2} = 81$$

$$\theta = \frac{3\pi}{2}$$



$$\bullet 3 \left( \cos \frac{3\pi}{8} + i \sin \frac{3\pi}{8} \right)$$

$$\bullet 3 \left( \cos \frac{7\pi}{8} + i \sin \frac{7\pi}{8} \right)$$

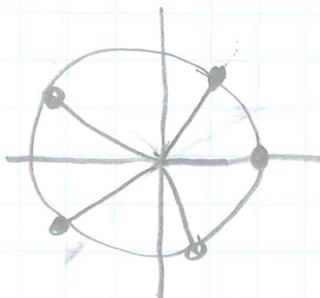
$$\bullet 3 \left( \cos \frac{11\pi}{8} + i \sin \frac{11\pi}{8} \right)$$

$$\bullet 3 \left( \cos \frac{15\pi}{8} + i \sin \frac{15\pi}{8} \right)$$

$$84) \sqrt[5]{}$$

$$r = \sqrt{32^2 + i0} = 32$$

$$\theta = 0$$



$$\sqrt[5]{32 \left( \cos \theta + i \sin \theta \right)}$$

$$= 32^{\frac{1}{5}} \left( \cos \frac{\theta + 2k\pi}{5} + i \sin \frac{\theta + 2k\pi}{5} \right)$$

$$\bullet 2 \left( \cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5} \right)$$

$$\bullet 2 \left( \cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5} \right)$$

$$\bullet 2 \left( \cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5} \right)$$

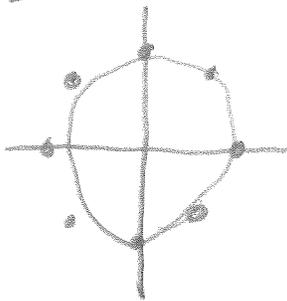
$$\bullet 2 \left( \cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5} \right)$$

$$\bullet 2 \left( \cos \theta + i \sin \theta \right) = 2$$

85)  $\sqrt[8]{1}$  

$r = \sqrt{1^2 + 0^2} = 1$

$\theta = 0$



$\sqrt[8]{1}(\cos \phi + i \sin \phi) = \cos \frac{\phi + 2k\pi}{8} - i \sin \frac{\phi + 2k\pi}{8}$

$\bullet \cos \phi + i \sin \phi = 1$

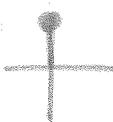
$\bullet \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

$\bullet \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} = i$

$\bullet \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

$\bullet$  etc

87)  $i^{1/3}$



$r = 1$

$\theta = \frac{\pi}{2}$

$= \cos \frac{\pi/2 + 2\pi k}{3} + i \sin \frac{\pi/2 + 2\pi k}{3}$

$\bullet \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

$\bullet \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}$

$\bullet$  etc

91)  $z^4 + 1 = 0$

$z^4 = -1, -1^{1/4}$



$r = 1$

$\theta = \pi$

$\frac{\pi + 2\pi k}{4}$

$\frac{\pi}{4} + k \cdot \frac{\pi}{2}$

$\bullet \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

$\bullet \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

$\bullet \cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

$\bullet \cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

92)  $z^8 = i$



$\theta = \frac{\pi}{2}$

$\frac{\pi + 2k\pi}{8}$

$\frac{\pi}{16} + \frac{k\pi}{4}$

94)  $z^6 - 1 = 0$

$z^6 = 1$



$\theta = \frac{\pi}{2}$

$\frac{\pi + 2k\pi}{6}$

$\frac{\pi}{12} + \frac{k\pi}{3}$