7.3 Exercises: p. 514; # 1, 3-5, 9-11, 15, 17, 19, 20, 23, 25, 27, 29, 37-39, 47, 48, 51, 55, 57, 61, 63, 69, 70, 72, 75

1) Double Angle Formula: $\sin 2x = 2\sin x \cos x$

3) $\sin x = \frac{5}{13}$ in Q1

$\cos x = \frac{12}{13}$

$\tan x = 5/12$

$\underline{\sin 2x} = 2\sin x \cos x$

$= 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right) = \boxed{\frac{120}{169}}$

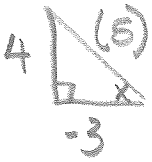
$\underline{\cos 2x} = \cos^2 x - \sin^2 x$

$= \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2 = \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$

$\underline{\tan 2x} = \frac{2\tan x}{1-\tan^2 x} = \frac{2(5/12)}{1-25/144} = \frac{10/12}{119/144}$

$= \boxed{\frac{120}{119}}$

4) $\tan x = -4/3$ in II



$\sin x = 4/5$

$\cos x = -3/5$

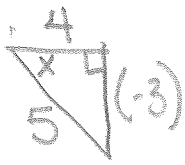
$\underline{\sin 2x} = 2(4/5)(-3/5) = \frac{-24}{25}$

$\underline{\cos 2x} = (-3/5)^2 - (4/5)^2 = \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$

$\underline{\tan 2x} = \frac{2(-4/3)}{1-(-4/3)^2} = \frac{-8/3}{1-16/9} = \frac{-8/3}{-7/9}$

$= 24/7$

5) $\cos x = 4/5$ $\csc \& \sin < 0$ in IV



$\sin x = -3/5$

$\tan x = -3/4$

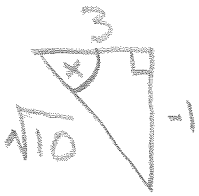
$\underline{\sin 2x} = 2(-3/5)(4/5) = \frac{-24}{25}$

$\underline{\cos 2x} = (4/5)^2 - (-3/5)^2$

$= 16/25 - 9/25 = 7/25$

$\underline{\tan 2x} = \frac{2(-3/4)}{1-(-3/4)^2} = \frac{-6/4}{1-9/16} = \frac{-6/4}{7/16} = \frac{-24}{7}$

9) $\tan x = -\frac{1}{3}$ $\cos x > 0$ in IV



$\sin x = -\frac{1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$

$\cos x = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$

$\underline{\sin 2x} = 2\left(-\frac{\sqrt{10}}{10}\right)\left(\frac{3\sqrt{10}}{10}\right)$

$= -\frac{60}{100} = \boxed{-3/5}$

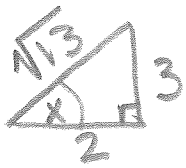
$\underline{\cos 2x} = \left(\frac{3\sqrt{10}}{10}\right)^2 - \left(-\frac{\sqrt{10}}{10}\right)^2 =$

$= \frac{90}{100} - \frac{10}{100} = \boxed{4/5}$

$\underline{\tan 2x} = \frac{2(-1/3)}{1-(-1/3)^2} = \frac{-2/3}{1-1/9} = \frac{-2/3}{8/9}$

$= -\frac{6}{8} = \boxed{-\frac{3}{4}}$

10) $\cot x = 2/3$ $\sin > 0$ in I



$\tan = 3/2$

$\sin = 3/\sqrt{13} = \frac{3\sqrt{13}}{13}$

$\cos = 2/\sqrt{13} = \frac{2\sqrt{13}}{13}$

$\underline{\sin 2x} = 2\left(\frac{3\sqrt{13}}{13}\right)\left(\frac{2\sqrt{13}}{13}\right)$

$= \boxed{12/13}$

$\underline{\cos 2x} = \left(\frac{2\sqrt{13}}{13}\right)^2 - \left(\frac{3\sqrt{13}}{13}\right)^2$

$= 4/13 - 9/13 = \boxed{-5/13}$

$\underline{\tan 2x} = \frac{2(3/2)}{1-(3/2)^2} = \frac{3}{1-9/4}$

$= \frac{3}{-5/4} = \boxed{-12/5}$

11) $\sin^4 x = \sin^2 x \cdot \sin^2 x = \frac{1-\cos 2x}{2} \cdot \frac{1-\cos 2x}{2}$

$= \frac{1 - 2\cos 2x + \cos^2 2x}{4}$

$\frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\cos^2 2x$

$\frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\left(\frac{1+\cos 4x}{2}\right) = \frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{8} + \frac{1}{8}\cos 4x$

$\boxed{\frac{1}{2}\left(\frac{3}{4} - \cos 2x + \frac{1}{4}\cos 4x\right)} \leftarrow \frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$

15) $\cos^4 x \cdot \sin^4 x = \cos^2 x \cdot \cos^2 x \cdot \sin^2 x \cdot \sin^2 x$

$\boxed{\text{oops 13 at end}}$ $= \left(\frac{1+\cos 2x}{2}\right)^2 \cdot \left(\frac{1-\cos 2x}{2}\right)^2$ re-order

$$\left(\frac{1+\cos 2x}{2} \cdot \frac{1-\cos 2x}{2}\right)^2 = \left(\frac{1-\cos^2 2x}{4}\right)^2 = \left(\frac{1}{4} - \frac{1}{4}\left(\frac{1+\cos 4x}{2}\right)\right)$$

$$\left(\frac{1}{8} - \frac{1}{8}\cos 4x\right)\left(\frac{1}{8} - \frac{1}{8}\cos 4x\right) \leftarrow \left(\frac{1}{4} - \frac{1}{8} - \frac{1}{8}\cos 4x\right)^2$$

FOIL

$$\frac{1}{64} - \frac{1}{32}\cos 4x + \frac{1}{64}\cos^2 4x \rightarrow \frac{1}{64} - \frac{1}{32}\cos 4x + \frac{1}{64}\left(\frac{1+\cos 8x}{2}\right)$$

$$\boxed{\frac{1}{32}\left(\frac{3}{4} - \cos 4x + \frac{1}{4}\cos 8x\right)} \leftarrow \frac{1}{64} - \frac{1}{32}\cos 4x + \frac{1}{128} + \frac{1}{128}\cos 8x$$

3/128

17) $\sin 15^\circ = \sin \frac{30^\circ}{2} = +\sqrt{\frac{1-\cos 30^\circ}{2}} = \sqrt{\frac{1-\sqrt{3}/2}{2}} = \sqrt{\frac{1}{2} - \frac{\sqrt{3}}{4}}$

in I, +

$$\boxed{\frac{1}{2}\sqrt{2-\sqrt{3}}} \leftarrow \sqrt{\frac{1}{4}(2-\sqrt{3})}$$

19) $\tan 22.5^\circ = \tan \frac{45^\circ}{2} = \frac{1-\cos 45^\circ}{\sin 45^\circ} = \frac{1-\frac{\sqrt{2}}{2}}{\sqrt{2}/2} = \frac{2-\sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$$\boxed{\sqrt{2}-1} \leftarrow \frac{2\sqrt{2}-2}{2}$$

20) $\sin 75^\circ = \sin \frac{150^\circ}{2} = +\sqrt{\frac{1-\cos 150^\circ}{2}} = \sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$

in I, +

$$\boxed{\frac{1}{2}\sqrt{2+\sqrt{3}}} \leftarrow \sqrt{\frac{1}{4}(2+\sqrt{3}} \leftarrow \sqrt{\frac{1}{2} + \frac{\sqrt{3}}{4}}$$

23) $\tan\frac{\pi}{8} = \tan\frac{\pi/4}{2} = \frac{1-\cos\pi/4}{\sin\pi/4}$ = same as #19

$\boxed{= \sqrt{2}-1}$

25) $\cos\frac{\pi}{12} = \cos\frac{\pi/6}{2} = +\sqrt{\frac{1+\cos\pi/6}{2}} = \sqrt{\frac{1+\sqrt{3}/2}{2}} = \sqrt{\frac{1}{2}+\frac{\sqrt{3}}{4}}$

in I, +

$\boxed{\frac{1}{2}\sqrt{2+\sqrt{3}}} \leftarrow \sqrt{\frac{1}{4}(2+\sqrt{3})}$

27) $\sin\frac{9\pi}{8} = \sin\frac{9\pi/4}{2} = -\sqrt{\frac{1-\cos 9\pi/4}{2}} = -\sqrt{\frac{1-\sqrt{2}/2}{2}}$

in III, −

$\boxed{-\frac{1}{2}\sqrt{2-\sqrt{2}}} \leftarrow -\sqrt{\frac{1}{4}(2-\sqrt{2})} \leftarrow -\sqrt{\frac{1}{2}-\frac{\sqrt{2}}{4}}$

29) a) $2\sin 18^\circ\cos 18^\circ = \sin(2\cdot 18^\circ) = \sin 36^\circ$

b) $2\sin 30^\circ\cos 30^\circ = \sin(2\cdot 30^\circ) = \sin 60^\circ$

37) $\sin x = 3/5$ in I

$\cos x = 4/5$

$\tan x = 3/4$

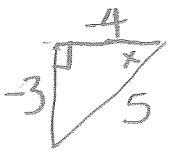
$\underline{\sin\frac{x}{2}} = +\sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1-4/5}{2}} = \boxed{\sqrt{\frac{1}{10}}}$

$\underline{\cos\frac{x}{2}} = +\sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1+4/5}{2}} = \boxed{\sqrt{\frac{9}{10}}}$

$\underline{\tan\frac{x}{2}} = \frac{1-\cos x}{\sin x} = \frac{1-4/5}{3/5} = \boxed{\frac{1}{3}}$

38) $\cos x = -4/5$ in III, $\frac{x}{2}$ is in II $\overset{+\sin}{-\cos,\tan}$

$\underline{\sin\frac{x}{2}} = +\sqrt{\frac{1-(-4/5)}{2}} = \boxed{\sqrt{\frac{9}{10}}}$



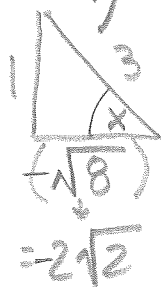
$\sin x = -3/5$

$\tan x = 3/4$

$\underline{\cos\frac{x}{2}} = -\sqrt{\frac{1+(-4/5)}{2}} = \boxed{-\sqrt{\frac{1}{10}}}$

$\underline{\tan\frac{x}{2}} = 1-(-4/5)/-3/5 = 9/-3 = \boxed{-3}$

39) $\csc x = 3$ in II, $\frac{x}{2}$ is in I



$\sin x = \frac{1}{3}$

$\cos x = \frac{2\sqrt{2}}{3}$

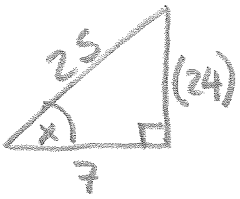
$$\sin\frac{x}{2} = +\sqrt{\frac{1+\frac{2\sqrt{2}}{3}}{2}\cdot\frac{3}{3}} = \boxed{\sqrt{\frac{3+2\sqrt{2}}{6}}} = \sqrt{\frac{1}{2}+\frac{\sqrt{2}}{3}}$$

$$\cos\frac{x}{2} = +\sqrt{\frac{1-\frac{2\sqrt{2}}{3}}{2}} = \sqrt{\frac{3-2\sqrt{2}}{6}}$$

$$\tan\frac{x}{2} = \frac{1+\frac{2\sqrt{2}}{3}}{1/3} = \boxed{3+2\sqrt{2}}$$

47) $\sin\left(2\cos^{-1}\frac{7}{25}\right) \rightarrow \sin 2x = 2\sin x\cos x$

$$= 2\left(\frac{24}{25}\right)\left(\frac{7}{25}\right) = \boxed{\frac{336}{625}}$$



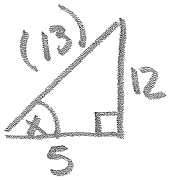
let $x = \cos^{-1}\frac{7}{25}$

so $\cos x = 7/25$

and $\sin x = 24/25$

48) $\cos\left(2\tan^{-1}\frac{12}{5}\right) \rightarrow \cos 2x = \cos^2 x - \sin^2 x$

$$= \frac{25}{169} - \frac{144}{169} = \boxed{\frac{-119}{169}}$$



let $x = \tan^{-1}\frac{12}{5}$

so $\sin x = 12/13$

and $\cos x = 5/13$

51) $\cos 2\theta$; $\sin\theta = -\frac{3}{5}$; in III

$\cos\theta = -\frac{4}{5}$

$$\cos 2\theta = \left(-\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$$

57) $\cos x \cdot \sin 4x = \frac{1}{2}\left[\sin(5x) - \sin(-3x)\right]$

61) $\sin 5x + \sin 3x = 2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2}$

$= 2\sin 4x \cos x$

63) $\cos 4x - \cos 6x = -2\sin 5x \cdot \sin -x$ even/odd

$= -2\sin 5x \cdot -\sin x$

$= 2\sin 5x \sin x$

69) $\cos 37.5^\circ \sin 7.5^\circ = \frac{1}{2}\left[\sin 45^\circ - \sin 30^\circ\right]$

$= \frac{1}{2}\left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right) = \frac{1}{4}\left(\sqrt{2} - 1\right)$

70) $\sin 75^\circ + \sin 15^\circ = 2\sin\frac{90^\circ}{2} \cdot \cos\frac{60^\circ}{2}$

$= 2\sin 45^\circ \cos 30^\circ$

$= 2 \cdot \sqrt{2}/2 \cdot \sqrt{3}/2 = \frac{\sqrt{6}}{2}$

72) $\cos\frac{\pi}{12} + \cos\frac{5\pi}{12} = 2\cos\frac{\overset{\pi/2}{6\pi/12}}{2}\cos\frac{\overset{-\pi/3}{-4\pi/12}}{2}$

$= 2\cos \pi/4 \cos -\pi/6$

$= 2 \cdot \sqrt{2}/2 \cdot \sqrt{3}/2 = \frac{\sqrt{6}}{2}$

75) $(\sin x + \cos x)^2 = 1 + \sin 2x$

FOIL

$\downarrow$

$\sin^2 x + 2\sin x\cos x + \cos^2 x$

$\sin^2 x + \cos^2 x + 2\sin x\cos x \rightarrow 1 + 2\sin x\cos x \rightarrow 1 + \sin 2x$

Pythag. Double Angle

Pythag!

13) $\cos^2 x \cdot \sin^4 x$

$$\left(\frac{1+\cos 2x}{2}\right)\left(\frac{1}{2}\left(\frac{3}{4} - \cos 2x + \frac{1}{4}\cos 4x\right)\right)$$

$$\frac{1}{2}\cdot\frac{1}{2}\left(1+\cos 2x\right)\left(\frac{3}{4} - \cos 2x + \frac{1}{4}\cos 4x\right)$$

$$\frac{1}{4}\left(\frac{3}{4} - \underline{\cos 2x} + \frac{1}{4}\cos 4x + \frac{3}{4}\cos 2x - \cos^2 2x + \frac{1}{4}\cos 2x \cos 4x\right)$$

combine

$$\frac{1}{4}\left(\frac{3}{4} - \frac{1}{4}\cos 2x + \frac{1}{4}\cos 4x - \cos^2 2x + \frac{1}{4}\cos 2x \cos 4x\right)$$

$$\cos^2 2x = \frac{1+\cos 2x}{2}$$

$$= \frac{1}{2} - \frac{1}{2}\cos 2x$$

$$\frac{1}{4}\left(\frac{1}{4} - \frac{3}{4}\cos 2x + \frac{1}{4}\cos 4x + \frac{1}{4}\cos 2x \cos 4x\right)$$

$$\frac{1}{4}\cdot\frac{1}{4}\left(1 - 3\cos 2x + \cos 4x + \cos 2x \cos 4x\right)$$

$\frac{1}{16}$

not sure why the difference from the book