5.1 Operations with Polynomials

Simplifying = rewriting expressions w/out parentheses or negative exponents

Monomials = 1 term all things connected with mult/Div

$-23a^2b^3c^4d^7$

polynomials
- $x+1$ binomial
- $x^2+12x+20$ trinomial
- more

Caveats exceptions: non-integer exponents on Variables

no radicals $\sqrt{x} = x^{\frac{1}{2}}$

no variables in a denominator $\frac{1}{x} = x^{-1}$

Degree of a polynomial: is the degree of its term with the highest degree

↑ exponent on a variable

$x^2 + 3x - 17$ $\quad 2^0$

$2^0 \quad 1^0 \quad 0^0$

const.

$3x^4 + 2x^9 - 7x^5 + 2$

$4^0 \quad 9^0 \quad 5^0 \quad 0^0$

$2x^3y^4 \quad 3+4=7^0$

Determine which of the following is a polynomial and find its degree.

a) $2x - 5y$ $\quad 1^0 \quad 1^0$ $\quad 1^0$

b) $\frac{1}{3}x^2 - 17$ $\quad 2^0$

c) $x^2 + 2x - x^{\frac{1}{2}}$ $\quad -\sqrt{x}$

d) $\frac{3x^2+2}{2y^3-3}$

e) $\sqrt{4x^3}$ $\quad 2x^3 \quad 3^0$

f) $\frac{x^3}{y}$

Power Rules

- product $x^a \cdot x^b = x^{a+b}$ $2^3 \cdot 2^4 = 2^{3+4} = 2^7$

$2 \cdot 2 \cdot 2 \times 2 \cdot 2 \cdot 2 \cdot 2 =$

- quotient $\frac{x^a}{x^b} = x^{a-b}$ $\frac{2^5}{2^2} = 2^{5-2} = 2^3$

- negative exponents (reciprocals)

$\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2} = 2^3$

$x^{-1} = \frac{1}{x}$ $\frac{1}{x^{-2}} = x^2$ $2^{-3} = \frac{1}{2^3}$ $\frac{1}{3^{-2}} = 3^2$

- power to a power $(x^2)^3 = x^{2 \cdot 3} = x^6$

$(2x^3)^4 = 2^4 x^{3 \cdot 4} = 2^4 x^{12} = 16x^{12}$

$\left(\frac{x^3}{y^4}\right)^2 = \frac{x^{3 \cdot 2}}{y^{4 \cdot 2}} = \frac{x^6}{y^8}$

$\left(\frac{a^2}{b^5}\right)^{-3} = \left(\frac{b^5}{a^2}\right)^3 = \frac{b^{5 \cdot 3}}{a^{2 \cdot 3}} = \frac{b^{15}}{a^6}$

$(3x^4y^7)(-2x^1y^{-4})$

$(3 \cdot -2)(x^4 \cdot x^1)(y^7 \cdot y^{-4})$

$-6x^5y^3$

$\frac{y^7}{y^4} = y^{7-4} =$

$\frac{15x^6y^8}{3x^7y^5} = \left(\frac{15}{3}\right)\left(\frac{x^6}{x^7}\right)\left(\frac{y^8}{y^5}\right)$

$5x^{-1}y^3 = \frac{5y^3}{x^1}$

$(3m^5n^{-3})^2$ $(3)^2(m^5)^2(n^{-3})^2$

$9m^{10}n^{-6}$ $\frac{9m^{10}}{n^6}$

$\left(\frac{3x^4 \cdot 2x^5}{4y^3}\right)^2 = \left(\frac{6x^9}{4y^3}\right)^2 = \frac{(6x^9)^2}{(4y^3)^2} = \frac{36x^{18}}{16y^6}$

or $\left(\frac{3x^9}{2y^3}\right)^2 = \frac{9x^{18}}{4y^6}$

Zero Powers

$3 \wedge 0 = 3^0 = 1$

$3\,y^x 0 = 1$

$3\,x^y 0 = 1$

$138 \wedge 0 \quad 138^0 = 1$

$x^0 = 1$

$(mnp)^0 = 1$

$0^0 = \text{undf}$

$-3^0 = (-1) \cdot 3^0 = (-1) \cdot 1 = -1$

$(-3)^0 = 1$

$$\left(\frac{(-73x^{14}y^{17}z^{43})^{18}}{(47x^{-3}y^{74}z^{-23})} \right)^0 = 1$$