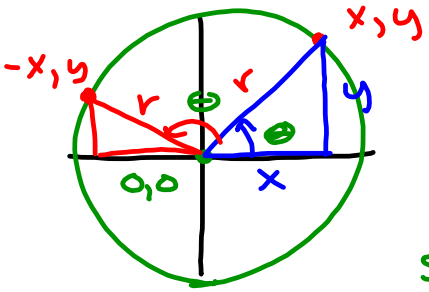
Section 4.3A: The Unit Circle

- The unit circle is used to extend the definitions of the 6 trig functions beyond acute ($<90°$) angles.



$r = \sqrt{x^2 + y^2}$

$r \geq 0$

r = hypotenuse

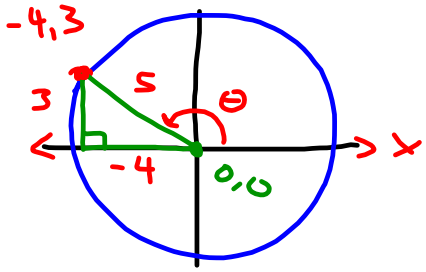
r is positive

$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$

a) $(-4,3)$ is a point on the terminal side of an angle in standard position. Find the value of the 6 trig functions.



$r = \sqrt{(-4)^2 + (3)^2} = \sqrt{16+9}$

$= \sqrt{25} = 5$

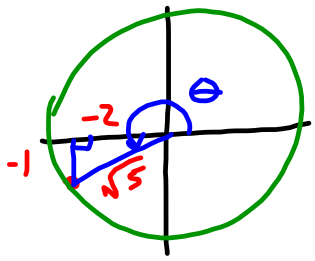
$\sin\theta = \frac{3}{5}$ $\csc\theta = \frac{5}{3}$

$\cos\theta = \frac{-4}{5}$ $\sec\theta = -\frac{5}{4}$

$\tan\theta = -\frac{3}{4}$ $\tan\theta = -\frac{4}{3}$

b) $(-2,-1)$

$r=\sqrt{(-2)^2+(-1)^2}=\sqrt{5}$

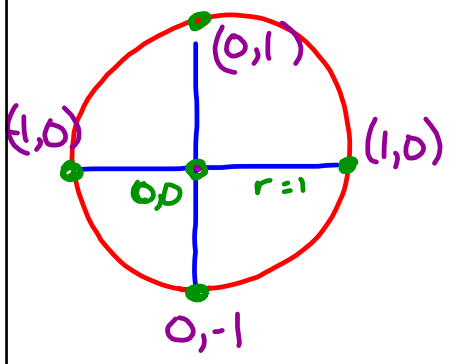


$\sin\theta=-\frac{1}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=-\frac{\sqrt{5}}{5}$ $\csc\theta=-\sqrt{5}$

$\cos\theta=\frac{-2\sqrt{5}}{5}$ $\sec\theta=-\frac{\sqrt{5}}{2}$

$\tan\theta=\frac{1}{2}$ $\cot\theta=2$

The Unit Circle ($r=1$)



$\sin\theta=y$ $\csc\theta=\frac{1}{y}$

$\cos\theta=x$ $\sec\theta=\frac{1}{x}$

$\tan\theta=\frac{y}{x}$ $\cot\theta=\frac{x}{y}$

$\sin 0=0$ $\csc 0=$ undefined

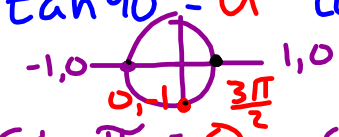
$\cos\theta=1$ $\sec 0=1$

$\tan 0=0$ $\cot 0=$ und

$\sin 90^\circ = 1$ $\csc 90^\circ = 1$

$\cos 90^\circ = 0$ $\sec 90^\circ = U$

$\tan 90^\circ = U$ $\cot 90^\circ = 0$



$\sin \pi = 0$ $\csc \pi = U$

$\cos \pi = -1$ $\sec \pi = -1$

$\tan \pi = 0$ $\cot \pi = U$

$\sin \frac{3\pi}{2} = -1$ $\csc \frac{3\pi}{2} = -1$

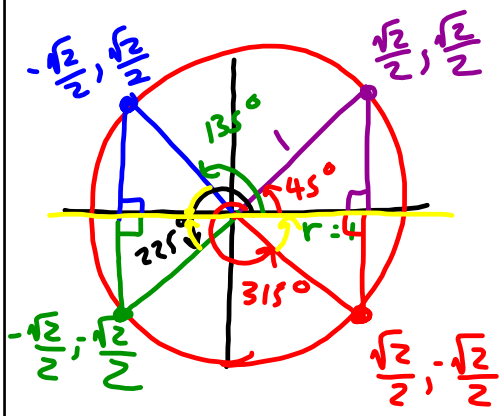
$\cos \frac{3\pi}{2} = 0$ $\sec \frac{3\pi}{2} = U$

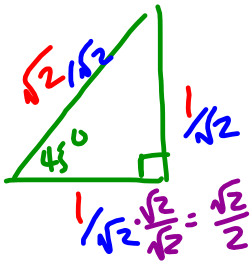
$\tan \frac{3\pi}{2} = U$ $\cot \frac{3\pi}{2} = 0$

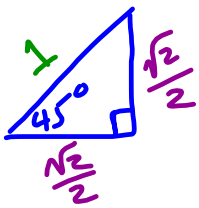
sin/cos/tan 360° is the same as sin/cos tan 0°

90°/450°/etc will also all be the same.

We can use this same Idea for other angles







Reference Angles θ'

angle to the x-axis

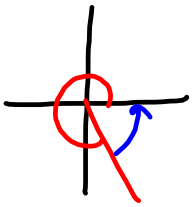
II: $\theta' = 180^\circ - \theta$

III: $\theta' = \theta - 180^\circ$

IV: $\theta' = 360^\circ - \theta$

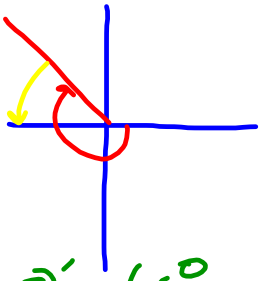
a) Find the Reference Angle:

300°



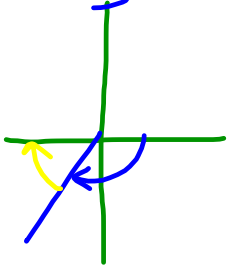
$\theta' = 60^\circ$

-240°



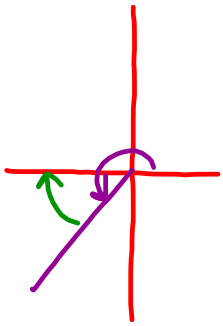
$\theta' = 60^\circ$

$-\frac{2\pi}{3}$



60°
$\left(\frac{\pi}{3}\right)$

$\frac{5\pi}{4}$



$\theta' = \frac{\pi}{4} = 45^\circ$

Signs of Reference Angles

+	+
−	−

$\sin = y$
$(\csc)$

−	+
−	+

$\cos = x$
$(\sec)$

−	+
+	−

$\tan = \frac{y}{x}$
$(\cot)$

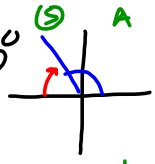
Sin	All
Tan	Cos

All Students Take Calc

Finding Trig Values:

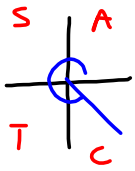
1) Find Reference Angle
2) Find Trig of Ref. Angle
3) Use Quadrant to determine ⊕/⊖ sign

$\cos 120^\circ$



$= -\cos 60^\circ = -\frac{1}{2}$

$\tan \frac{7\pi}{4}$



$= -\tan \frac{\pi}{4} = -1$

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