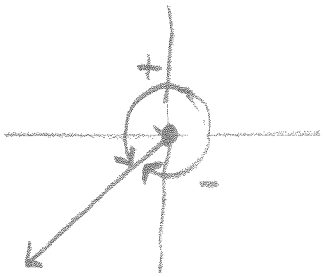
4.2B: Coterminal Angles, Arc Length, Speed, Sectors

Coterminal Angles: (Angles that have the same starting and ending rays)



- Degrees θ & $\theta + n360^\circ$ n is any +/- integer

$225^\circ + 360^\circ = 585^\circ$

$+ 720^\circ = 945^\circ$

$- 360^\circ = -135^\circ$

$- 720^\circ = -495^\circ$

etc

- Michael Jackson 720°
- Tony Hawk 900°
- Shawn White 1080°

- Radians θ & $\theta + 2n\pi$

ex: Find the coterminal angles and draw 1θ & -1θ

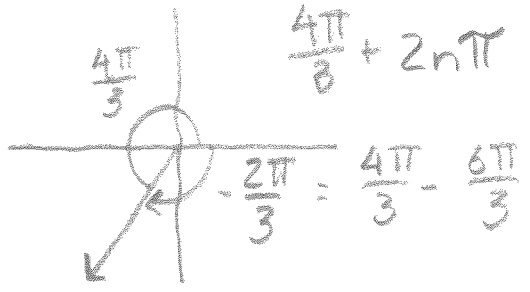
$\frac{5\pi}{4} + \left(\frac{8\pi}{4}\right) = \frac{13\pi}{4}$ (2π)

$\frac{5\pi}{4} + \frac{16\pi}{4} = \frac{21\pi}{4}$

$\frac{5\pi}{4} - \frac{8\pi}{4} = -\frac{3\pi}{4}$

$\frac{5\pi}{4} - \frac{16\pi}{4} = -\frac{11\pi}{4}$

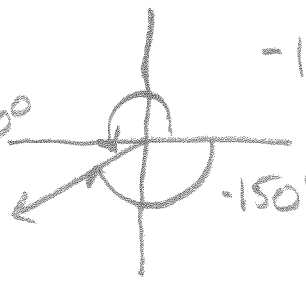
a) $\frac{4\pi}{3}$



$\frac{4\pi}{3} + 2n\pi$

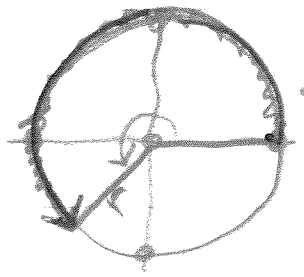
b) -150°

$150^\circ + 360^\circ = 210^\circ$



$-150^\circ + n360^\circ$

Arc Length (s) = $r\theta$ (θ is always in Radians)



a) find the length of the arc intercepted by a central angle of $\frac{5\pi}{4}$ with a radius of 6cm

$s = \frac{5\pi}{4} \cdot 6 = \frac{30\pi}{4} = 7\frac{1}{2}\pi = 23.56$cm

b) find the length of the arc with $\theta = 720°$ and an $r = 10$m

(much of the time Arc lengths include multiple rotations $\times 2\pi$)

$s = 720° \cdot 10 = 4\pi(10) = 40\pi$
$= 125.66$m

c) Wheel has a radius of 1ft. How many miles will you travel in 1,000,000 rotations?

$\theta = 1{,}000{,}000 \times 2\pi \times 1\text{ft} = 2{,}000{,}000\pi\text{ ft}$
$= 6{,}283{,}185.307\text{ ft}$
$\div 5280 = 1190$ miles

Linear Speed & Angular Speed

Angular Speed → rate that an object rotates.

Linear Speed → Rate an object moves along a circular path

Linear Speed (v): $v = \frac{s}{t}$ ← arc length moved / ← time

Angular Speed (ω): $\omega = \frac{\theta}{t}$ ← angle rotated / ← time

a) A DVD with diameter = 120mm (r = 60mm)
if it rotates @ 3.5 rps what is its angular speed

$$\omega = \frac{\theta}{t} = \frac{3.5 \cdot 2\pi}{1} = 7\pi \text{ rps} \approx 22 \text{ rps}$$

b) If it begins to overheat in your X-box and slows to 3 rps, what is the Linear Speed in meters/min

$$v = \frac{s}{t} = \frac{r\theta}{t} = \frac{60\text{mm} \cdot (3 \cdot 2\pi)}{1} : \frac{360\pi \text{ mm}}{1 \text{ s}} \cdot \frac{60 \text{ s}}{1 \text{ min}} \cdot \frac{1 \text{ meter}}{1000 \text{ mm}} = 21.6\pi \text{ m/min}$$

$$67.86 \text{ m/min}$$

Sector Area (Area of part of a circle) (A)

$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{A}{\pi r^2} \overset{\text{proportional}}{=} \frac{\text{Arc length}}{\text{circumference}} = \frac{r\theta}{2\pi r}$$

Also: $\pi r^2\left(\frac{\theta}{360^\circ}\right)$ or $\pi r^2\left(\frac{\theta}{2\pi}\right)$

$$\frac{A}{\pi r^2} = \frac{r\theta}{2\pi r} \cdot \pi r \qquad A = \frac{1}{2} r^2 \theta$$

a) let $\theta = \frac{3\pi}{4}$ (135°) $r = 1.5$ ft $\quad A = \frac{1}{2}(1.5)^2 \cdot \frac{3\pi}{4} = 0.84375\pi$

$$= 2.65 \text{ ft}^2$$

b) 

windshield wiper
what is the area it wipes?

$$130 \cdot \frac{\pi}{180} = \frac{13\pi}{18}$$

$$A = \frac{1}{2}(26)^2 \cdot \frac{13\pi}{18} = 244.1\pi$$

$$= 766.9 \text{ in}^2$$