LESSON 2-6

Ratios and Proportions

fractions!

Then

You evaluated percents by using a proportion.

Now

1 Compare ratios. $\frac{1}{2}$

2 Solve proportions. $\frac{1}{2} = \frac{3}{6}$

Why?

Ratios allow us to compare many items by using a common reference. The table below shows the number of restaurants a certain popular fast food chain has per 10,000 people in the United States as well as other countries. This allows us to compare the number of these restaurants using an equal reference.

Countries	United States	New Zealand	Canada	Australia	Japan	Singapore
Number of Restaurants per 10,000 People	0.433	0.369	0.352	0.349	0.282	0.273

Proportion - Ratio = Ratio

1 Ratios and Proportions

The comparison between the number of restaurants and the number of people is a ratio. A **ratio** is a comparison of two numbers by division. The ratio of x to y can be expressed in the following ways.

1 to 2 $\quad x$ to y $\qquad$ 1:2 $\quad x : y$ $\qquad \frac{x}{y}$ $\quad \frac{1}{2}$

Suppose you wanted to determine the number of restaurants per 100,000 people in Australia. Notice that this ratio is equal to the original ratio.

$$\frac{0.349 \text{ restaurants}}{10{,}000 \text{ people}} = \frac{3.49 \text{ restaurants}}{100{,}000 \text{ people}}$$

× 10 × 10

An equation stating that two ratios are equal is called a **proportion**. So, we can state that $\frac{0.349}{10{,}000} = \frac{3.49}{100{,}000}$ is a proportion.

Example 1 Determine Whether Ratios Are Equivalent

Determine whether $\frac{2}{3}$ and $\frac{16}{24}$ are equivalent ratios. Write *yes* or *no*. Justify your answer.

$$\frac{2}{3} \overset{\div 1}{=} \frac{2}{3} \qquad \frac{2}{3}\ \frac{16}{24} \overset{\div 8}{=} \frac{2}{3}$$

When expressed in simplest form, the ratios are equivalent.

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Determine whether each pair of ratios are equivalent ratios. Write *yes* or *no*. Justify your answer.

1A. $\frac{6}{10}, \frac{2}{5}$

$\frac{3}{5} \neq \frac{2}{5}$

1B. $\frac{1}{6}, \frac{5}{30}$ $\frac{1}{6}$

$\frac{1}{6} = \frac{1}{6}$ ✓

StudyTip

Means and Extremes

To solve a proportion using cross products, write an equation that sets the product of the extremes equal to the product of the means.

There are special names for the terms in a proportion.

1.5 and 1.2 are called the means. They are the middle terms of the proportion.

$$0.2 : 1.5 = 1.2 : 9.0$$

0.2 and 9.0 are called the extremes. They are the first and last terms of the proportion.

KeyConcept Means-Extremes Property of Proportion

Words	In a proportion, the product of the extremes is equal to the product of the means.
Symbols	If $\frac{a}{b} = \frac{c}{d}$ and $b, d \neq 0$, then $ad = bc$.
Examples	Since $\frac{2}{4} = \frac{1}{2}$, $2(2) = 4(1)$ or $4 = 4$.

Example 2 Cross Products

Use cross products to determine whether each pair of ratios forms a proportic

a. $\frac{2}{3.5}, \frac{8}{14}$

$\frac{2}{3.5} \stackrel{?}{=} \frac{8}{14}$	Original proportion
$2(14) \stackrel{?}{=} 3.5(8)$	Cross products
$28 = 28$ ✓	Simplify.

The cross products are equal, so the ratios form a proportion.

cross products

$2 \cdot 14 = 3.5 \cdot 8$

$28 = 28$ ✓

b. $\frac{0.3}{1.5}, \frac{0.5}{2.0}$

$\frac{0.3}{1.5} \stackrel{?}{=} \frac{0.5}{2.0}$	Original proportion
$0.3(2.0) \stackrel{?}{=} 1.5(0.5)$	Cross products
$0.6 \neq 0.75$ ✗	Simplify.

The cross products are not equal, so the ratios do not form a proportion.

$0.3 \cdot 2.0 = 1.5 \cdot 5$

$0.6 \neq 0.75$

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2A. $\frac{0.2}{1.8}, \frac{1}{0.9}$

$0.2 \cdot 0.9 = 1 \cdot 1.8$

$\times 1.8$

$0.18 \neq 1.8$

2B. $\frac{3}{8}, \frac{21}{56}$

$3 \cdot 56 = 21 \cdot 8$

$168 = 168$ ✓

StudyTip

Cross Products When you find cross products, you are said to be *cross multiplying*.

2 Solve Proportions

To solve proportions, use cross products.

Example 3 Solve a Proportion

Solve each proportion. If necessary, round to the nearest hundredth.

a. $\frac{x}{10} = \frac{3}{5}$

$\frac{x}{10} = \frac{3}{5}$	Original proportion
$x(5) = 10(3)$	Find the cross products.
$5x = 30$	Simplify.
$\frac{5x}{5} = \frac{30}{5}$	Divide each side by 5.
$x = 6$	Simplify.

$\frac{x}{10} = \frac{3}{5}$

$\frac{5x}{5} = \frac{30}{5}$

$x = 6$

b. $\frac{x-2}{14} = \frac{2}{7}$

$\frac{x-2}{14} = \frac{2}{7}$	Original proportion
$(x-2)7 = 14(2)$	Find the cross products.
$7x - 14 = 28$	Simplify.
$7x = 42$	Add 14 to each side.
$x = 6$	Divide each side by 7.

$\frac{x-2}{14} = \frac{2}{7}$

$7(x-2) = 2 \cdot 14$

$7x - 14 = 28$

$+14 \quad +14$

$\frac{7x}{7} = \frac{42}{7}$

$x = 6$

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3A. $\frac{r}{8}, \frac{25}{40}$

$40r = 8 \cdot 25$

$\frac{40r}{40} = \frac{200}{40}$

$r = 5$

3B. $\frac{x+4}{5}, \frac{3}{8}$

$8(x+4) = 3 \cdot 5$

$8x + 32 = 15$

$-32 \quad -32$

$\frac{8x}{8} = \frac{-17}{8}$

$x = -2.125$

The ratio of two measurements having different units of measure is called a **rate**. For example, a price of \$9.99 per 10 songs is a rate. A rate that tells how many of one item is being compared to 1 of another item is called a **unit rate**.

Real-World Example 4 Rate of Growth

RETAIL **In the past two years, a retailer has opened 232 stores. If the rate of growth remains constant, how many stores will the retailer open in the next 3 years?**

Understand Let r represent the number of retail stores.

Plan Write a proportion for the problem.

$$\frac{232 \text{ retail stores}}{2 \text{ years}} = \frac{r \text{ retail stores}}{3 \text{ years}}$$

Solve

$\frac{232}{2} = \frac{r}{3}$	Original proportion
$232(3) = 2r$	Find the cross products.
$696 = 2r$	Simplify.
$\frac{696}{2} = \frac{2r}{2}$	Divide each side by 2.
$348 = r$	Simplify.

The retailer will open 348 stores in 3 years.

A rate called a **scale** is used to make a **scale model** of something too large or too small to be convenient at actual size.

Real-World Example 5 Scale and Scale Models

MOUNTAIN TRAIL **The Ramsey Cascades Trail is about $1\frac{1}{8}$ inches long on a map with scale 3 inches = 10 miles. What is the actual length of the trail?**

Let ℓ represent the actual length.

scale → $\frac{3}{10} = \frac{1\frac{1}{8}}{\ell}$ ← scale
actual → ← actual

$3(\ell) = 1\frac{1}{8}(10)$	Find the cross products.
$3\ell = \frac{45}{4}$	Simplify.
$3\ell \div 3 = \frac{45}{4} \div 3$	Divide each side by 3.
$\ell = \frac{15}{4}$ or $3\frac{3}{4}$	Simplify.

The actual length is about $3\frac{3}{4}$ miles.

Guided Practice

5. **AIRPLANES** On a model airplane, the scale is 5 centimeters = 2 meters. If the model's wingspan is 28.5 centimeters, what is the actual wingspan?

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–2 Determine whether each pair of ratios are equivalent ratios. Write *yes* or *no*.

1. $\frac{3}{7}, \frac{9}{14}$
2. $\frac{7}{8}, \frac{42}{48}$
3. $\frac{2.8}{4.4}, \frac{1.4}{2.1}$

Example 3 Solve each proportion. If necessary, round to the nearest hundredth.

4. $\frac{n}{9} = \frac{6}{27}$
5. $\frac{4}{u} = \frac{28}{35}$
6. $\frac{3}{8} = \frac{b}{10}$

Example 4

7. **RACE** Jennie ran the first 6 miles of a marathon in 58 minutes. If she is able to maintain the same pace, how long will it take her to finish the 26.2 miles?

Example 5

8. CCSS **PRECISION** On a map of North Carolina, Raleigh and Asheville are about 8 inches apart. If the scale is 1 inch = 12 miles, how far apart are the cities?

Check Your Understanding

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Examples 1–2 Determine whether each pair of ratios are equivalent ratios. Write *yes* or *no*.

1. $\frac{3}{7}, \frac{9}{14}$ **no**
2. $\frac{7}{8}, \frac{42}{48}$ **yes**
3. $\frac{2.8}{4.4}, \frac{1.4}{2.1}$ **no**

Example 3 Solve each proportion. If necessary, round to the nearest hundredth.

4. $\frac{n}{9} = \frac{6}{27}$ **2**
5. $\frac{4}{u} = \frac{28}{35}$ **5**
6. $\frac{3}{8} = \frac{b}{10}$ **3.75**

Example 4

7. **RACE** Jennie ran the first 6 miles of a marathon in 58 minutes. If she is able to maintain the same pace, how long will it take her to finish the 26.2 miles?

≈ 253.3 min or ≈ 4 h 13.3 min

Example 5

8. CCSS **PRECISION** On a map of North Carolina, Raleigh and Asheville are about 8 inches apart. If the scale is 1 inch = 12 miles, how far apart are the cities? **about 96 mi**