LESSON 2-1

Power and Radical Functions

Then

- You analyzed parent functions and their families of graphs. (Lesson 1-5)

Now

1. Graph and analyze power functions. 2^x
2. Graph and analyze radical functions, and solve radical equations.

Why?

- Suspension bridges are used to span long distances by hanging, or suspending, the main deck using steel cables. The amount of weight that a steel cable can support is a function of the cable's diameter and can be modeled by a power function.

$\sqrt[3]{x}$ or $x^{\frac{1}{3}}$ ← rational Exp.

radical → $\sqrt{\ }$

Review Vocabulary

Reciprocal Functions
Reciprocal functions have the form $f(x) = \frac{a}{x}$ (Lesson 1-5)

Recall that $f(x) = \frac{1}{x}$ or x^{-1} is undefined at $x = 0$. Similarly, $f(x) = x^{-2}$ and $f(x) = x^{-3}$ are undefined at $x = 0$. Because power functions can be undefined when $n < 0$, the graphs of these functions will contain discontinuities.

even powers

odd powers

Example 2 Functions with Negative Exponents

Graph and analyze each function. Describe the domain, range, intercepts, end behavior, continuity, and where the function is increasing or decreasing.

a. $f(x) = 3x^{-2} = \frac{3}{x^2}$

x	−3	−2	−1	0	1	2	3
$f(x)$	$0.\overline{3}$	0.75	3	undefined	3	0.75	$0.\overline{3}$

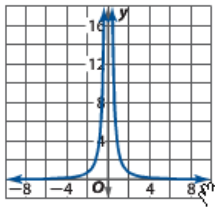
Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(0, \infty)$

Intercepts: none $x \neq 0$

End behavior: $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = 0$

Continuity: infinite discontinuity at $x = 0$

Increasing: $(-\infty, 0)$ Decreasing: $(0, \infty)$



b. $f(x) = -\frac{3}{4}x^{-5}$ $= -\frac{3}{4} \cdot \frac{1}{x^5} = -\frac{3}{4x^5}$

x	−3	−2	−1	0	1	2	3
$f(x)$	0.0031	0.0234	0.75	undefined	−0.75	−0.0234	−0.0031

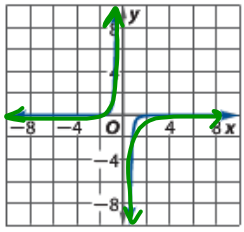
Domain: $(-\infty, 0) \cup (0, \infty)$ $x \neq 0$ Range: $(-\infty, 0) \cup (0, \infty)$

Intercepts: none

End behavior: $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = 0$

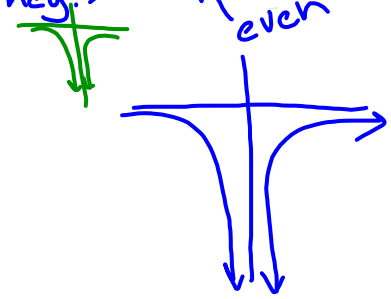
Continuity: infinite discontinuity at $x = 0$

Increasing: $(-\infty, 0)$ and $(0, \infty)$

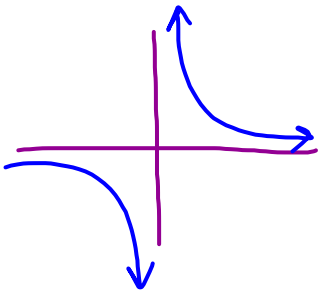


GuidedPractice

2A. $f(x) = -\frac{1}{2}x^{-4}$ neg. even



2B. $f(x) = 4x^{-3}$ + odd



ReviewVocabulary

Rational Exponents exponents written as fractions in simplest form. (Lesson 0-4)

Recall that $x^{\frac{1}{n}}$ indicates the nth root of x, and $x^{\frac{p}{n}}$, where $\frac{p}{n}$ is in simplest form, indicates the nth root of x^p. If n is an even integer, then the domain must be restricted to nonnegative values.

Example 3 Rational Exponents

Graph and analyze each function. Describe the domain, range, intercepts, end behavior, continuity, and where the function is increasing or decreasing.

a. $f(x) = x^{\frac{5}{2}}$ $= \sqrt{x^5}$

even root (denominator) ½ graph

x	0	1	2	3	4	5	6
$f(x)$	0	1	5.657	15.588	32	55.902	88.182

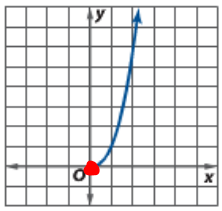
Domain: $[0, \infty)$ Range: $[0, \infty)$

x- and y-Intercepts: 0

End behavior: $\lim_{x \to \infty} f(x) = \infty$

Continuity: continuous on $[0, \infty)$

Increasing: $(0, \infty)$



b. $f(x) = 6x^{-\frac{2}{3}}$ ← even odd root

$6x^{-\frac{5}{3}}$

x	−3	−2	−1	0	1	2	3
$f(x)$	2.884	3.780	6	undefined	6	3.780	2.884

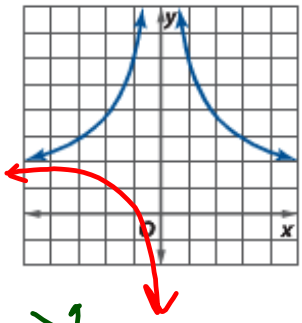
Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(0, \infty)$

Intercepts: none

End behavior: $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = 0$

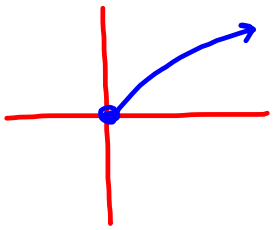
Continuity: infinite discontinuity at $x = 0$

Increasing: $(-\infty, 0)$ Decreasing: $(0, \infty)$

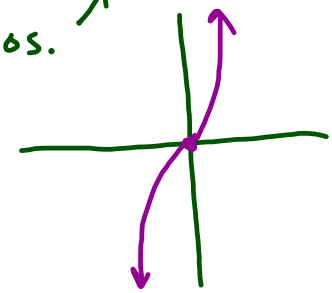


GuidedPractice

3A. $f(x) = 2x^{\frac{3}{4}}$ below 1, even



3B. $f(x) = 10x^{\frac{5}{3}}$ odd, >1, pos.



2 Radical Functions

An expression with rational exponents can be written in radical form.

Exponential Form		Radical Form
$x^{\frac{p}{n}}$	=	$\sqrt[n]{x^p}$

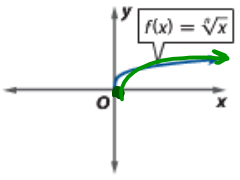
Power functions with rational exponents represent the most basic of radical functions. A **radical function** is a function that can be written as $f(x) = x^{\frac{p}{n}}$, where n and p are positive integers greater than 1 that have no common factors. Some examples of radical functions are shown below.

$f(x) = 3\sqrt{5x^3}$ $f(x) = -5\sqrt[3]{x^4 + 3x^2 - 1}$ $f(x) = \sqrt[4]{x + 12} + \frac{1}{2}x - 7$

KeyConcept Radical Functions

Let f be the radical function $f(x) = \sqrt[n]{x}$ where n is a positive integer.

n Even



Domain and Range: $[0, \infty)$

***x*- and *y*-Intercept:** 0

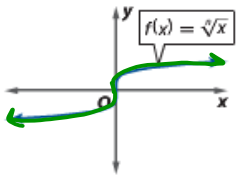
Continuity: continuous on $[0, \infty)$

Symmetry: none **Increasing:** $(0, \infty)$

Extrema: absolute minimum at (0, 0)

End Behavior: $\lim_{x \to \infty} f(x) = \infty$

n Odd



Domain and Range: $(-\infty, \infty)$

***x*- and *y*-Intercept:** 0

Continuity: continuous on $(-\infty, \infty)$

Symmetry: origin **Increasing:** $(-\infty, \infty)$

Extrema: none

End Behavior: $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$

Example 5 Graph Radical Functions

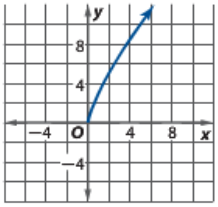
even

Graph and analyze each function. Describe the domain, range, intercepts, end behavior, continuity, and where the function is increasing or decreasing.

WatchOut!
Radical Functions Remember that when n is even, the domain and range will have restrictions.

a. $f(x) = 2\sqrt[5]{5x^3}$ $\quad 2\boxed{(5)x^3}^{\frac{1}{4}}$ odd

x	0	1	2	3	4	5
$f(x)$	0	2.99	5.03	6.82	8.46	10



Domain and Range: $[0, \infty)$
x- and y-Intercepts: 0
End behavior: $\lim_{x\to\infty} f(x) = \infty$
Continuity: continuous on $[0, \infty)$
Increasing: $(0, \infty)$

$5x^3$
$5 \cdot x \cdot x \cdot x$

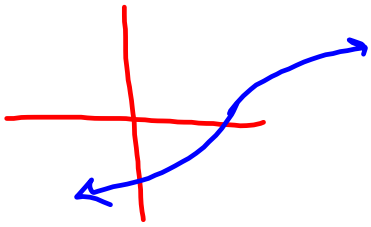
$+ < 1$

odd

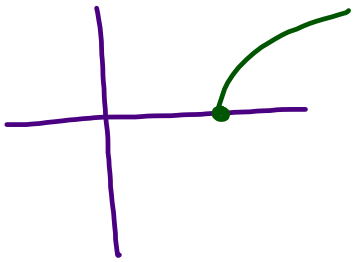
even

GuidedPractice

5A. $f(x) = \frac{1}{4}\sqrt[5]{6x - 8}$



5B. $f(x) = \frac{1}{2}\sqrt[4]{2x^3 - 16}$



Like radical functions, a radical equation is any equation in which a variable is in the radicand. To solve a radical equation, first isolate the radical expression. Then raise each side of the equation to a power equal to the index of the radical to eliminate the radical.

Raising each side of an equation to a power sometimes produces **extraneous solutions**, or solutions that do not satisfy the original equation. It is important to check for extraneous solutions.

Example 6 Solve Radical Equations

Solve each equation.

a. $2x = \sqrt{100 - 12x} - 2$

$2x = \sqrt{100 - 12x} - 2$	Original equation
$(2x + 2)^2 = \sqrt{100 - 12x}^{\,2}$	Isolate the radical.
$4x^2 + 8x + 4 = 100 - 12x$	Square each side to eliminate the radical.
$4x^2 + 20x - 96 = 0$	Subtract 100 − 12x from each side.
$4(x^2 + 5x - 24) = 0$	Factor.
$4(x + 8)(x - 3) = 0$	Factor.
$x + 8 = 0$ or $x - 3 = 0$	Zero Product Property
$x = -8$ $\quad$ $x = 3$	Solve.
~~CHECK $x = -8$~~	CHECK $x = 3$

1) isolate the radical

$(2x+2)^2 = (2x+2)(2x+2)$

$4x^2 + 4x + 4x + 4$

$2(-8) = \sqrt{100 - 12(-8)} - 2$

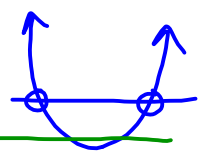
$-16 = \sqrt{196} - 2$

$-16 \neq 14 - 2$

$2(3) = \sqrt{100 - 12(3)} - 2$

$6 = \sqrt{64} - 2$

$6 = 8 - 2$ ✓



b. $\sqrt[3]{(x-5)^2}+14=50$

$\sqrt[3]{(x-5)^2}+14=50$	Original equation
$\sqrt[3]{(x-5)^2}=36$	Isolate the radical.
$(x-5)^2=46{,}656$	Raise each side to the third power. (The index is 3.)
$x-5=\pm 216$	Take the square root of each side.
$x=221$ or -211	Add 5 to each side.

A check of the solutions in the original equation confirms that the solutions are valid.

WatchOut!

Squaring Radical Expressions

Take extra care as you square $5-\sqrt{15-x}$. While similar to using the FOIL method with binomial expressions, there are some differences. Be sure to account for every term.

c. $\sqrt{x-2}=5-\sqrt{15-x}$

$\sqrt{x-2}=5-\sqrt{15-x}$	Original equation
$x-2=25-10\sqrt{15-x}+(15-x)$	Square each side.
$2x-42=-10\sqrt{15-x}$	Isolate the radical.
$4x^2-168x+1764=100(15-x)$	Square each side.
$4x^2-168x+1764=1500-100x$	Distributive Property
$4x^2-68x+264=0$	Combine like terms.
$4(x^2-17x+66)=0$	Factor.
$4(x-6)(x-11)=0$	Factor.
$x-6=0$ or $x-11=0$	Zero Product Property
$x=6$ $\quad x=11$	Solve.

$(5-\sqrt{15-x})(5-\sqrt{15-x})$

$25-5\sqrt{15-x}-5\sqrt{15-x}+15-x$

$40-10\sqrt{15-x}-x$

Guided Practice

6A. $3x = 3 + \sqrt{18x - 18}$ **6B.** $\sqrt[3]{4x + 8} + 3 = 7$ **6C.** $\sqrt{x + 7} = 3 + \sqrt{2 - x}$

$x+7=(3+\sqrt{2-x})(3+\sqrt{2-x})$

$x+7=9+6\sqrt{2-x}+2-x$

$+x \quad -11 \quad -11 \quad +x$

$2x-4=6\sqrt{2-x}$

$\div 2$

$(x-2)(x-2) \leftarrow (x-2)^2=(3\sqrt{2-x})^2$

$x^2-4x+4=9(2-x)$

$x^2-4x+4=18-9x$

$+9x \quad -18 \quad -18 \quad +9x$

$x^2+5x-14=0$

$(x+7)(x-2)=0$

$x=-7 \quad x=2$

p. 92: 1-15 odd
19-29 odd
45-55 odd
70-73 all