

Chapter 6 Review:

Attached are the Chapter Review and the last year's Chapter 6 Test. I'll post the key to the test on the website.

The Concept Check does a good job of listing the concepts in Ch.6. The formulas should be in the class notes. Given a lot of time, you might explore the Chapter Review first, and then test yourself with the Chapter Test. I would be willing to answer questions on both on Thursday, as well as on any of the quizzes.

CHAPTER 6 | REVIEW

■ CONCEPT CHECK

- (a) Explain the difference between a positive angle and a negative angle.
(b) How is an angle of measure 1 degree formed?
(c) How is an angle of measure 1 radian formed?
(d) How is the radian measure of an angle θ defined?
(e) How do you convert from degrees to radians?
(f) How do you convert from radians to degrees?
- (a) When is an angle in standard position?
(b) When are two angles coterminal?
- (a) What is the length s of an arc of a circle with radius r that subtends a central angle of θ radians?
(b) What is the area A of a sector of a circle with radius r and central angle θ radians?
- If θ is an acute angle in a right triangle, define the six trigonometric ratios in terms of the adjacent and opposite sides and the hypotenuse.
- What does it mean to solve a triangle?
- If θ is an angle in standard position, $P(x, y)$ is a point on the terminal side, and r is the distance from the origin to P , write expressions for the six trigonometric functions of θ .
- Which trigonometric functions are positive in Quadrants I, II, III, and IV?
- If θ is an angle in standard position, what is its reference angle $\bar{\theta}$?
- (a) State the reciprocal identities.
(b) State the Pythagorean identities.
- (a) What is the area of a triangle with sides of length a and b and with included angle θ ?
(b) What is the area of a triangle with sides of length a , b , and c ?
- Define the inverse sine function \sin^{-1} . What are its domain and range?
- Define the inverse cosine function \cos^{-1} . What are its domain and range?
- Define the inverse tangent function \tan^{-1} . What are its domain and range?
- (a) State the Law of Sines.
(b) State the Law of Cosines.
- Explain the ambiguous case in the Law of Sines.

■ EXERCISES

1–2 ■ Find the radian measure that corresponds to the given degree measure.

- (a) 60° (b) 330° (c) -135° (d) -90°
- (a) 24° (b) -330° (c) 750° (d) 5°

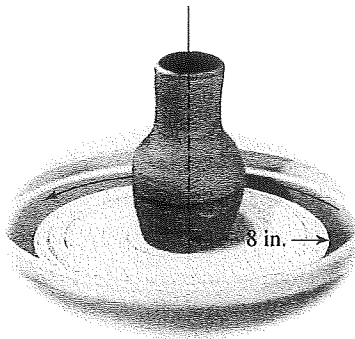
3–4 ■ Find the degree measure that corresponds to the given radian measure.

- (a) $\frac{5\pi}{2}$ (b) $-\frac{\pi}{6}$ (c) $\frac{9\pi}{4}$ (d) 3.1

- (a) 8 (b) $-\frac{5}{2}$ (c) $\frac{11\pi}{6}$ (d) $\frac{3\pi}{5}$

- Find the length of an arc of a circle of radius 8 m if the arc subtends a central angle of 1 rad.
- Find the measure of a central angle θ in a circle of radius 5 ft if the angle is subtended by an arc of length 7 ft.
- A circular arc of length 100 ft subtends a central angle of 70° . Find the radius of the circle.
- How many revolutions will a car wheel of diameter 28 in. make over a period of half an hour if the car is traveling at 60 mi/h?

9. New York and Los Angeles are 2450 mi apart. Find the angle that the arc between these two cities subtends at the center of the earth. (The radius of the earth is 3960 mi.)
10. Find the area of a sector with central angle 2 rad in a circle of radius 5 m.
11. Find the area of a sector with central angle 52° in a circle of radius 200 ft.
12. A sector in a circle of radius 25 ft has an area of 125 ft^2 . Find the central angle of the sector.
13. A potter's wheel with radius 8 in. spins at 150 rpm. Find the angular and linear speeds of a point on the rim of the wheel.



14. In an automobile transmission a *gear ratio* g is the ratio

$$g = \frac{\text{angular speed of engine}}{\text{angular speed of wheels}}$$

The angular speed of the engine is shown on the tachometer (in rpm).

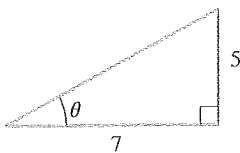
A certain sports car has wheels with radius 11 in. Its gear ratios are shown in the following table. Suppose the car is in fourth gear and the tachometer reads 3500 rpm.

- (a) Find the angular speed of the engine.
- (b) Find the angular speed of the wheels.
- (c) How fast (in mi/h) is the car traveling?

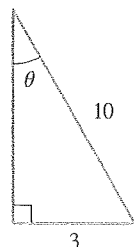
Gear	Ratio
1st	4.1
2nd	3.0
3rd	1.6
4th	0.9
5th	0.7

- 15–16 ■ Find the values of the six trigonometric ratios of θ .

15.

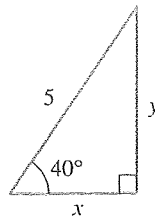


16.

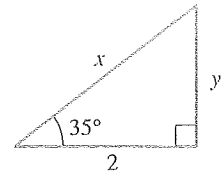


- 17–20 ■ Find the sides labeled x and y , rounded to two decimal places.

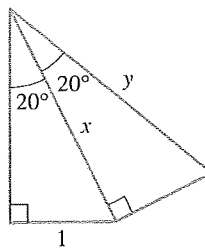
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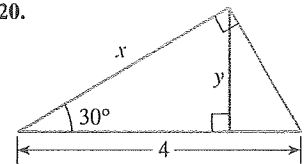
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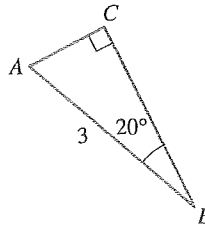


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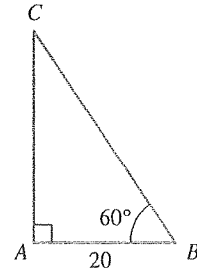


- 21–24 ■ Solve the triangle.

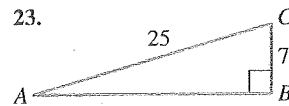
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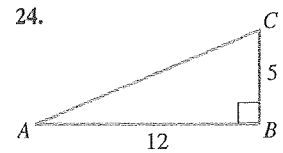
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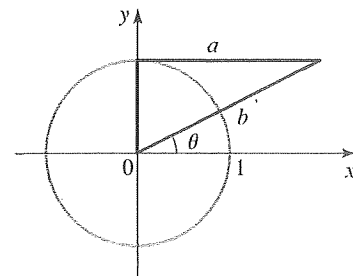
23.



24.



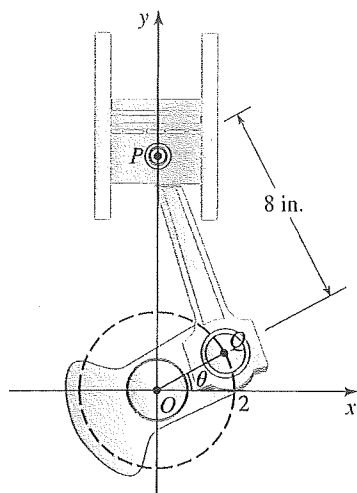
25. Express the lengths a and b in the figure in terms of the trigonometric ratios of θ .



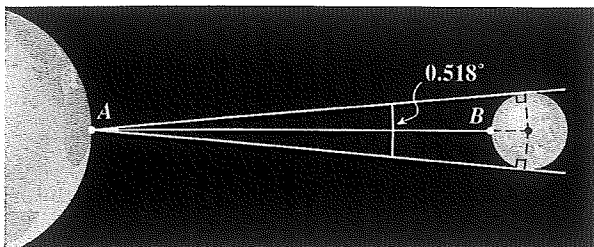
26. The highest free-standing tower in North America is the CN Tower in Toronto, Canada. From a distance of 1 km from its base, the angle of elevation to the top of the tower is 28.81° . Find the height of the tower.

27. Find the perimeter of a regular hexagon that is inscribed in a circle of radius 8 m.

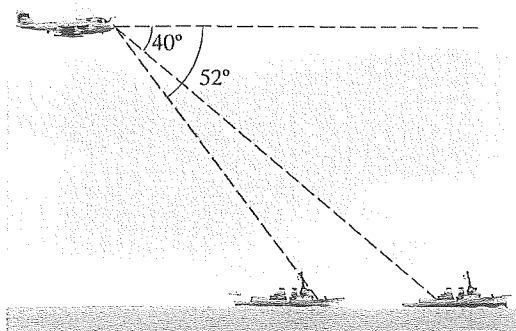
28. The pistons in a car engine move up and down repeatedly to turn the crankshaft, as shown. Find the height of the point P above the center O of the crankshaft in terms of the angle θ .



29. As viewed from the earth, the angle subtended by the full moon is 0.518° . Use this information and the fact that the distance AB from the earth to the moon is 236,900 mi to find the radius of the moon.



30. A pilot measures the angles of depression to two ships to be 40° and 52° (see the figure). If the pilot is flying at an elevation of 35,000 ft, find the distance between the two ships.



- 31–42 ■ Find the exact value.

31. $\sin 315^\circ$ 32. $\csc \frac{9\pi}{4}$
 33. $\tan(-135^\circ)$ 34. $\cos \frac{5\pi}{6}$

35. $\cot\left(-\frac{22\pi}{3}\right)$ 36. $\sin 405^\circ$

37. $\cos 585^\circ$ 38. $\sec \frac{22\pi}{3}$

39. $\csc \frac{8\pi}{3}$ 40. $\sec \frac{13\pi}{6}$

41. $\cot(-390^\circ)$ 42. $\tan \frac{23\pi}{4}$

43. Find the values of the six trigonometric ratios of the angle θ in standard position if the point $(-5, 12)$ is on the terminal side of θ .
44. Find $\sin \theta$ if θ is in standard position and its terminal side intersects the circle of radius 1 centered at the origin at the point $(-\sqrt{3}/2, 1/2)$.
45. Find the acute angle that is formed by the line $y - \sqrt{3}x + 1 = 0$ and the x -axis.
46. Find the six trigonometric ratios of the angle θ in standard position if its terminal side is in Quadrant III and is parallel to the line $4y - 2x - 1 = 0$.

- 47–50 ■ Write the first expression in terms of the second, for θ in the given quadrant.

47. $\tan \theta$, $\cos \theta$; θ in Quadrant II

48. $\sec \theta$, $\sin \theta$; θ in Quadrant III

49. $\tan^2 \theta$, $\sin \theta$; θ in any quadrant

50. $\csc^2 \theta \cos^2 \theta$, $\sin \theta$; θ in any quadrant

- 51–54 ■ Find the values of the six trigonometric functions of θ from the information given.

51. $\tan \theta = \sqrt{7}/3$, $\sec \theta = \frac{4}{3}$ 52. $\sec \theta = \frac{41}{40}$, $\csc \theta = -\frac{41}{9}$

53. $\sin \theta = \frac{3}{5}$, $\cos \theta < 0$ 54. $\sec \theta = -\frac{13}{5}$, $\tan \theta > 0$

55. If $\tan \theta = -\frac{1}{2}$ for θ in Quadrant II, find $\sin \theta + \cos \theta$.

56. If $\sin \theta = \frac{1}{2}$ for θ in Quadrant I, find $\tan \theta + \sec \theta$.

57. If $\tan \theta = -1$, find $\sin^2 \theta + \cos^2 \theta$.

58. If $\cos \theta = -\sqrt{3}/2$ and $\pi/2 < \theta < \pi$, find $\sin 2\theta$.

- 59–62 ■ Find the exact value of the expression.

59. $\sin^{-1}(\sqrt{3}/2)$ 60. $\tan^{-1}(\sqrt{3}/3)$

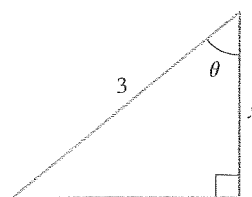
61. $\tan(\sin^{-1} \frac{2}{5})$ 62. $\sin(\cos^{-1} \frac{3}{8})$

- 63–64 ■ Rewrite the expression as an algebraic expression in x .

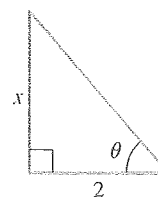
63. $\sin(\tan^{-1} x)$ 64. $\sec(\sin^{-1} x)$

- 65–66 ■ Express θ in terms of x .

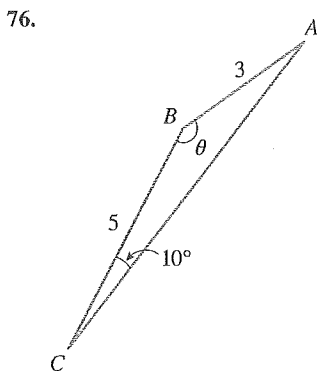
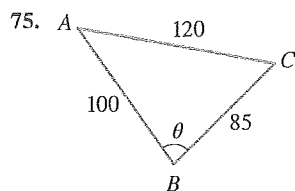
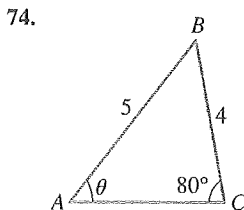
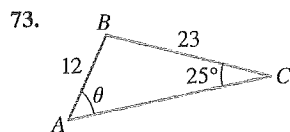
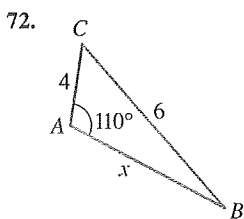
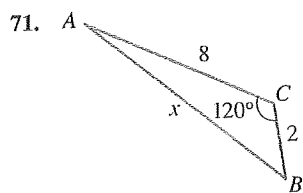
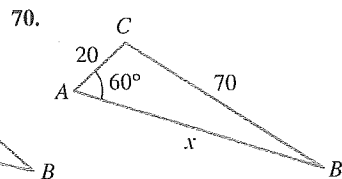
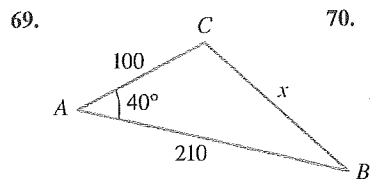
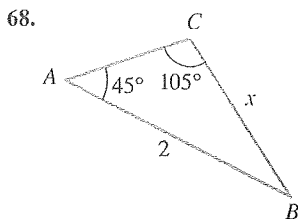
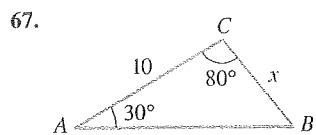
65.



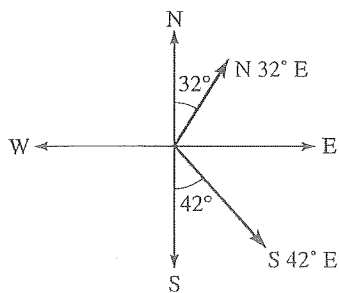
66.



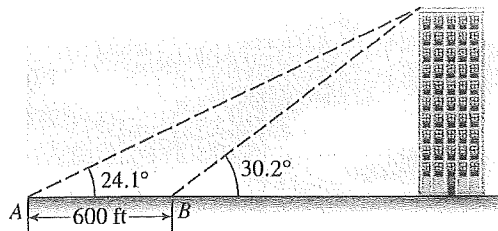
67–76 ■ Find the side labeled x or the angle labeled θ .



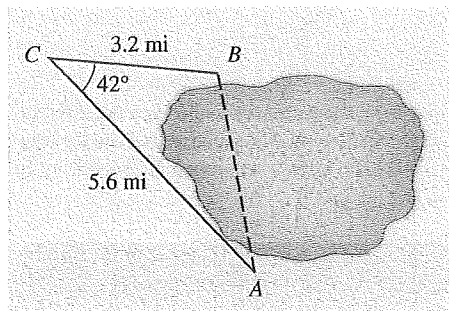
77. Two ships leave a port at the same time. One travels at 20 mi/h in a direction $N 32^\circ E$, and the other travels at 28 mi/h in a direction $S 42^\circ E$ (see the figure). How far apart are the two ships after 2 h?



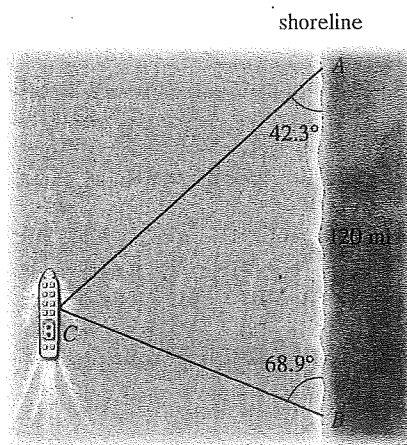
78. From a point A on the ground, the angle of elevation to the top of a tall building is 24.1° . From a point B , which is 600 ft closer to the building, the angle of elevation is measured to be 30.2° . Find the height of the building.



79. Find the distance between points A and B on opposite sides of a lake from the information shown.



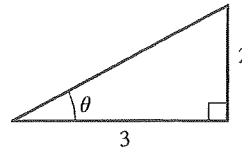
80. A boat is cruising the ocean off a straight shoreline. Points A and B are 120 mi apart on the shore, as shown. It is found that $\angle A = 42.3^\circ$ and $\angle B = 68.9^\circ$. Find the shortest distance from the boat to the shore.



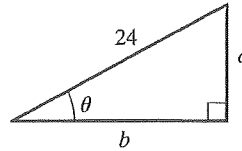
81. Find the area of a triangle with sides of length 8 and 14 and included angle 35° .

82. Find the area of a triangle with sides of length 5, 6, and 8.

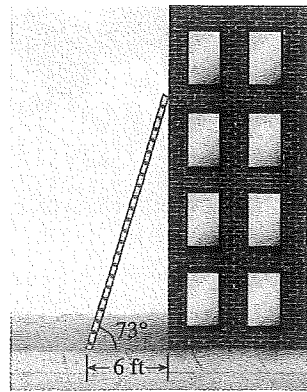
- Find the radian measures that correspond to the degree measures 330° and -135° .
- Find the degree measures that correspond to the radian measures $\frac{4\pi}{3}$ and -1.3 .
- The rotor blades of a helicopter are 16 ft long and are rotating at 120 rpm.
 - Find the angular speed of the rotor.
 - Find the linear speed of a point on the tip of a blade.
- Find the exact value of each of the following.
 - $\sin 405^\circ$
 - $\tan(-150^\circ)$
 - $\sec \frac{5\pi}{3}$
 - $\csc \frac{5\pi}{2}$
- Find $\tan \theta + \sin \theta$ for the angle θ shown.



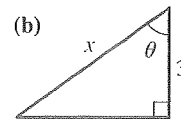
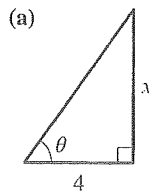
- Express the lengths a and b shown in the figure in terms of θ .



- If $\cos \theta = -\frac{1}{3}$ and θ is in Quadrant III, find $\tan \theta \cot \theta + \csc \theta$.
- If $\sin \theta = \frac{5}{13}$ and $\tan \theta = -\frac{5}{12}$, find $\sec \theta$.
- Express $\tan \theta$ in terms of $\sec \theta$ for θ in Quadrant II.
- The base of the ladder in the figure is 6 ft from the building, and the angle formed by the ladder and the ground is 73° . How high up the building does the ladder touch?

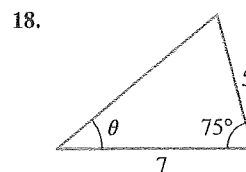
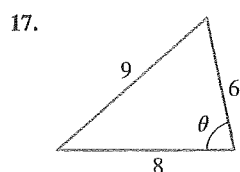
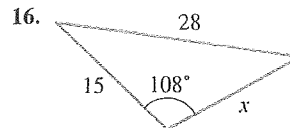
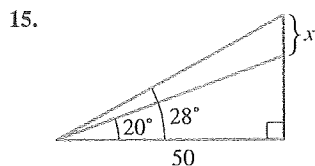
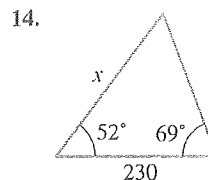
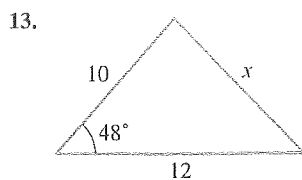


- Express θ in each figure in terms of x .



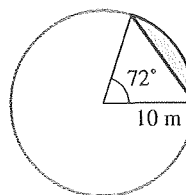
- Find the exact value of $\cos(\tan^{-1} \frac{9}{40})$.

13–18 ■ Find the side labeled x or the angle labeled θ .



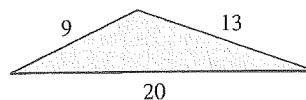
19. Refer to the figure below.

- (a) Find the area of the shaded region.
- (b) Find the perimeter of the shaded region.

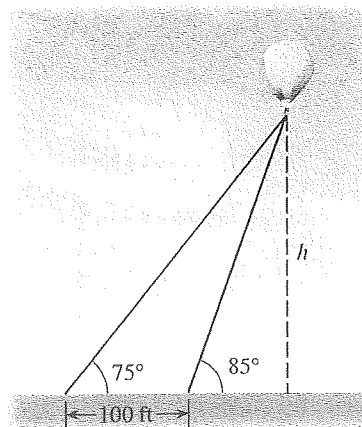


20. Refer to the figure below.

- (a) Find the angle opposite the longest side.
- (b) Find the area of the triangle.



21. Two wires tether a balloon to the ground, as shown. How high is the balloon above the ground?



13. $\angle A \approx 48^\circ$, $\angle B \approx 79^\circ$, $c \approx 3.2$
 15. $\angle A \approx 50^\circ$, $\angle B \approx 73^\circ$, $\angle C \approx 57^\circ$
 17. $\angle A_1 \approx 83.6^\circ$, $\angle C_1 \approx 56.4^\circ$, $a_1 \approx 193$;
 $\angle A_2 \approx 16.4^\circ$, $\angle C_2 \approx 123.6^\circ$, $a_2 \approx 54.9$ 19. No such triangle
 21. 2 23. 25.4 25. 89.2° 27. 24.3 29. 54 31. 26.83
 33. 5.33 35. 40.77 37. 3.85 cm² 39. 2.30 mi 41. 23.1 mi
 43. 2179 mi 45. (a) 62.6 mi (b) S 18.2° E 47. 96°
 49. 211 ft 51. 3835 ft 53. \$165,554

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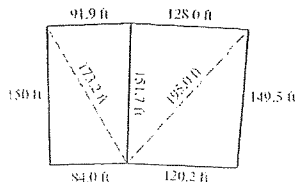
1. (a) $\pi/3$ (b) $11\pi/6$ (c) $-3\pi/4$ (d) $-\pi/2$
 3. (a) 450° (b) -30° (c) 405° (d) $(558/\pi)^\circ \approx 177.6^\circ$
 5. 8 m 7. 82 ft 9. 0.619 rad $\approx 35.4^\circ$ 11. 18,151 ft²
 13. 300π rad/min ≈ 942.5 rad/min,
 7539.8 in./min $= 628.3$ ft/min
 15. $\sin \theta = 5/\sqrt{74}$, $\cos \theta = 7/\sqrt{74}$, $\tan \theta = \frac{5}{7}$,
 $\csc \theta = \sqrt{74}/5$, $\sec \theta = \sqrt{74}/7$, $\cot \theta = \frac{7}{5}$
 17. $x \approx 3.83$, $y \approx 3.21$ 19. $x \approx 2.92$, $y \approx 3.11$
 21. $A = 70^\circ$, $a \approx 2.819$, $b \approx 1.026$
 23. $A = 16.3^\circ$, $C \approx 73.7^\circ$, $c = 24$
 25. $a = \cot \theta$, $b = \csc \theta$ 27. 48 m 29. 1076 mi 31. $-\sqrt{2}/2$
 33. 1 35. $-\sqrt{3}/3$ 37. $-\sqrt{2}/2$ 39. $2\sqrt{3}/3$ 41. $-\sqrt{3}$
 43. $\sin \theta = \frac{12}{13}$, $\cos \theta = -\frac{5}{13}$, $\tan \theta = -\frac{12}{5}$,
 $\csc \theta = \frac{13}{12}$, $\sec \theta = -\frac{13}{5}$, $\cot \theta = -\frac{5}{12}$ 45. 60°
 47. $\tan \theta = -\sqrt{1 - \cos^2 \theta}/\cos \theta$
 49. $\tan^2 \theta = \sin^2 \theta/(1 - \sin^2 \theta)$
 51. $\sin \theta = \sqrt{7}/4$, $\cos \theta = \frac{3}{4}$, $\csc \theta = 4\sqrt{7}/7$, $\cot \theta = 3\sqrt{7}/7$
 53. $\cos \theta = -\frac{4}{5}$, $\tan \theta = -\frac{3}{4}$, $\csc \theta = \frac{5}{3}$, $\sec \theta = -\frac{5}{4}$, $\cot \theta = -\frac{4}{3}$
 55. $-\sqrt{5}/5$ 57. 1 59. $\pi/3$ 61. $2/\sqrt{21}$ 63. $x/\sqrt{1+x^2}$
 65. $\theta = \cos^{-1}(x/3)$ 67. 5.32 69. 148.07 71. 9.17
 73. 54.1° or 125.9° 75. 80.4° 77. 77.3 mi 79. 3.9 mi
 81. 32.12

CHAPTER 6 TEST ■ PAGE 487

1. $11\pi/6$, $-3\pi/4$ 2. 240° , -74.5°
 3. (a) 240π rad/min ≈ 753.98 rad/min
 (b) $12,063.7$ ft/min $= 137$ mi/h 4. (a) $\sqrt{2}/2$
 (b) $\sqrt{3}/3$ (c) 2 (d) 1 5. $(26 + 6\sqrt{13})/39$
 6. $a = 24 \sin \theta$, $b = 24 \cos \theta$ 7. $(4 - 3\sqrt{2})/4$
 8. $-\frac{12}{13}$ 9. $\tan \theta = -\sqrt{\sec^2 \theta - 1}$ 10. 19.6 ft
 11. (a) $\theta = \tan^{-1}(x/4)$ (b) $\theta = \cos^{-1}(3/x)$ 12. $\frac{40}{41}$
 13. 9.1 14. 250.5 15. 8.4 16. 19.5 17. 78.6° 18. 40.2°
 19. (a) 15.3 m² (b) 24.3 m 20. (a) 129.9° (b) 44.9
 21. 554 ft

FOCUS ON MODELING ■ PAGE 490

1. 1.41 mi 3. 14.3 m 5. (c) 2349.8 ft
 7.


CHAPTER 7
SECTION 7.1 ■ PAGE 498

1. all; 1 2. $\cos(-x) = \cos x$ 3. $\sin t$ 5. $\tan \theta$ 7. -1
 9. $\csc u$ 11. $\tan \theta$ 13. 1 15. $\cos y$ 17. $\sin^2 x$ 19. $\sec x$
 21. $2 \sec u$ 23. $\cos^2 x$ 25. $\cos \theta$
 27. (a) $\text{LHS} = \frac{1 - \sin^2 x}{\sin x} = \text{RHS}$
 29. $\text{LHS} = \sin \theta \frac{\cos \theta}{\sin \theta} = \text{RHS}$
 31. $\text{LHS} = \cos u \frac{1}{\cos u} \cot u = \text{RHS}$
 33. $\text{LHS} = \sin B + \cos B \frac{\cos B}{\sin B}$
 $= \frac{\sin^2 B + \cos^2 B}{\sin B} = \frac{1}{\sin B} = \text{RHS}$
 35. $\text{LHS} = -\frac{\cos \alpha}{\sin \alpha} \cos \alpha - \sin \alpha = \frac{-\cos^2 \alpha - \sin^2 \alpha}{\sin \alpha}$
 $= \frac{-1}{\sin \alpha} = \text{RHS}$
 37. $\text{LHS} = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}$
 $= \frac{1}{\cos \theta \sin \theta} = \text{RHS}$
 39. $\text{LHS} = 1 - \cos^2 \beta = \sin^2 \beta = \text{RHS}$
 41. $\text{LHS} = \frac{(\sin x + \cos x)^2}{(\sin x + \cos x)(\sin x - \cos x)} = \frac{\sin x + \cos x}{\sin x - \cos x}$
 $= \frac{(\sin x + \cos x)(\sin x - \cos x)}{(\sin x - \cos x)(\sin x - \cos x)} = \text{RHS}$
 43. $\text{LHS} = \frac{\frac{1}{\cos t} - \cos t}{\frac{1}{\cos t}} \cdot \frac{\cos t}{\cos t} = \frac{1 - \cos^2 t}{1} = \text{RHS}$
 45. $\text{LHS} = \frac{1}{\cos^2 y} = \sec^2 y = \text{RHS}$
 47. $\text{LHS} = \cot x \cos x + \cot x - \csc x \cos x - \csc x$
 $= \frac{\cos^2 x}{\sin x} + \frac{\cos x}{\sin x} - \frac{\cos x}{\sin x} - \frac{1}{\sin x} = \frac{\cos^2 x - 1}{\sin x}$
 $= \frac{-\sin^2 x}{\sin x} = \text{RHS}$
 49. $\text{LHS} = \sin^2 x \left(1 + \frac{\cos^2 x}{\sin^2 x} \right) = \sin^2 x + \cos^2 x = \text{RHS}$
 51. $\text{LHS} = 2(1 - \sin^2 x) - 1 = 2 - 2\sin^2 x - 1 = \text{RHS}$
 53. $\text{LHS} = \frac{1 - \cos \alpha}{\sin \alpha} \cdot \frac{1 + \cos \alpha}{1 + \cos \alpha}$
 $= \frac{1 - \cos^2 \alpha}{\sin \alpha(1 + \cos \alpha)} = \frac{\sin^2 \alpha}{\sin \alpha(1 + \cos \alpha)} = \text{RHS}$
 55. $\text{LHS} = \frac{\sin^2 \theta}{\cos^2 \theta} - \frac{\sin^2 \theta \cos^2 \theta}{\cos^2 \theta}$
 $= \frac{\sin^2 \theta(1 - \cos^2 \theta)}{\cos^2 \theta} = \frac{\sin^2 \theta \sin^2 \theta}{\cos^2 \theta} = \text{RHS}$
 57. $\text{LHS} = \frac{\sin x - 1}{\sin x + 1} \cdot \frac{\sin x + 1}{\sin x + 1} = \frac{\sin^2 x - 1}{(\sin x + 1)^2} = \text{RHS}$

Eastern Oregon University Concurrent Enrollment/Credit by Proficiency Program

Math 112, Spring, 2014

Exam 1

name: _____

Show any relevant work. For each problem, circle your answer

1. (12 points) For each angle below, change degrees to radians and radians to degrees:

a. $5\pi/6$ rad

b. -300°

c. $7\pi/2$ rad

d. 765°

2. (6 points each) Find the exact value of each of the following. Include a circle sketch and reference triangle.

a. $\tan(2\pi/3)$

b. $\sin(-7\pi/6)$

c. $\sec \frac{5\pi}{4}$

d. $\cos(\sin^{-1} \frac{2}{3})$

e. $\sin^{-1}(-\frac{1}{2})$

3. (10 points) A sector of a circle with a central angle measure of $2\pi/3$ rad has area of 5m^2 . Find the radius of the circle.

4. (12 points) A ponderosa pine casts a shadow of 240 ft on level ground when the angle of elevation to the sun is 24° . How tall is the tree?

5. (12 points) If $\cos \theta = 3/8$ and $\sin \theta < 0$, find the values of the other five elementary trigonometric functions at θ . Draw a sketch with appropriate reference angle.

a. $\sin \theta =$

b. $\sec \theta =$

c. $\csc \theta =$

d. $\tan \theta =$

e. $\cot \theta =$

6. (12 points) Solve each triangle below. Sketch appropriate triangles labeled such that side a is opposite angle A , side b opposite angle B , and side c opposite angle C .

a. $a = 4$, $c = 9$, $B = 65^\circ$

b. $A = 75^\circ$, $B = 60^\circ$, $c = 340$ miles

7. (12 points) A commercial jet leaves Kuala Lumpur and flies on a bearing of N 30° E for 90 minutes. The pilot then turns to fly on a bearing of N 90° W and flies on this bearing for 4 hours. If the plane is flying at a constant speed of 500 miles per hour, how far is it from Kuala Lumpur at this time?