

Eastern Oregon University  
Concurrent Enrollment/Credit by Proficiency Program

Math 112, Spring, 2015

Exam 3

name/school: Key

Show any relevant work. For each problem, circle your answer.

1. (20 points) Verify each of the following identities:

10 a.  $\sec^2 x - \tan^2 x = 1$

$$\begin{aligned} \sec^2 x - \tan^2 x &= \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} \quad \boxed{+6} \\ &= \frac{1 - \sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} = 1 \end{aligned}$$

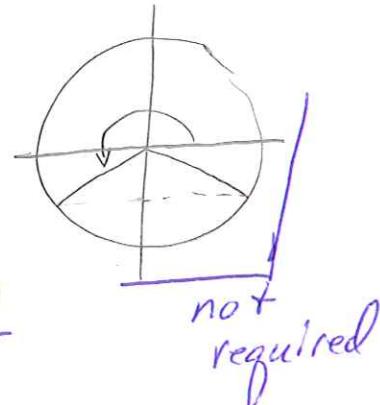
10 b.  $\sin(\frac{\pi}{2} - x) = \cos x$  (Use a sum or difference formula.)

$$\begin{aligned} \sin(\frac{\pi}{2} - x) &= \underbrace{\sin \frac{\pi}{2} \cdot \cos x}_{+2} - \underbrace{\cos \frac{\pi}{2} \cdot \sin x}_{+2} \quad \boxed{+2} \\ &= 1 \cdot \cos x - 0 \cdot \sin x \quad \boxed{+4} \\ &= \cos x \end{aligned}$$

2. (16 points) Find all solutions to each equation in the interval  $0 \leq \theta \leq 2\pi$ :

8 a.  $\cos \theta(2 \sin \theta + 1) = 0$

$$\begin{aligned} \cos \theta &= 0 \quad \boxed{+2} & \text{or} \quad 2 \sin \theta + 1 &= 0 \quad \boxed{+2} \\ \theta &= \frac{\pi}{2} \text{ or } \frac{3\pi}{2} \quad \boxed{-} & \sin \theta &= -\frac{1}{2} \quad \boxed{+2} \\ & & \theta &= \frac{7\pi}{6} \text{ or } \frac{11\pi}{6} \quad \boxed{+2} \end{aligned}$$



8 b.  $4 \sin^2 \theta - 3 = 0$

$$\begin{aligned} 4 \sin^2 \theta &= 3 \\ \sin^2 \theta &= \frac{3}{4} \quad \boxed{+2} \\ \sin \theta &= \pm \frac{\sqrt{3}}{2} \quad \boxed{+2} \end{aligned}$$

$$\rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \text{ or } \frac{5\pi}{3}$$

+4

3. (24 points) Use addition or subtraction formulas, double-angle or half-angle formulas as appropriate to evaluate each of the following expressions.

$$6 \quad a. \cos \frac{5\pi}{12}, \quad \frac{5\pi}{12} = \frac{8\pi}{12} - \frac{3\pi}{12} \quad \boxed{+2}$$

$$= \frac{2\pi}{3} - \frac{\pi}{4}$$

$$\cos \frac{5\pi}{12} = \cos \left( \frac{2\pi}{3} - \frac{\pi}{4} \right)$$

$$+2 \quad [ = \cos \frac{2\pi}{3} \cdot \cos \frac{\pi}{4} + \sin \frac{2\pi}{3} \cdot \sin \frac{\pi}{4}$$

$$+2 \quad [ = -\frac{1}{2} \left( \frac{\sqrt{2}}{2} \right) + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{(\sqrt{3}-1)\sqrt{2}}{4}$$

or  $\frac{5\pi}{12} = \frac{5\pi/6}{2} \quad \boxed{+2}$   
use  $\cos \frac{x}{2}$  with  $x = \frac{5\pi}{6}$   
:

← other simplifications  
are possible

Suppose  $\cos x = -\frac{3}{7}$  and  $x$  is a quadrant II angle. Find each of the following:

$$6 \quad b. \cos 2x = \cos^2 x - \sin^2 x \quad \boxed{OR}$$

$$+2 \quad [ = \left( -\frac{3}{7} \right)^2 - \left( \frac{\sqrt{40}}{7} \right)^2 = 2\cos^2 x - 1$$

$$= \frac{9}{49} - \frac{40}{49} = 2\left(\frac{9}{49}\right) - 1$$

$$= -\frac{31}{49} = \frac{18}{49} - \frac{49}{49}$$

$$+2 \quad [ = -\frac{31}{49}$$

$$6 \quad c. \sin 2x$$

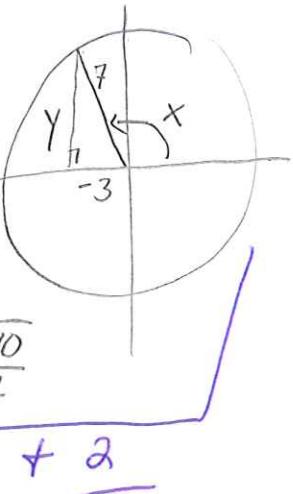
$$y^2 = 7^2 - (-3)^2$$

$$= 49 - 9$$

$$= 40$$

$$y = \sqrt{40}$$

$$\sin x = \frac{\sqrt{40}}{7}$$



$$= 2 \sin x \cdot \cos x \quad \boxed{+2}$$

$$= 2 \frac{\sqrt{40}}{7} \cdot \left( -\frac{3}{7} \right) = -\frac{6\sqrt{40}}{49}$$

$$\boxed{+2} \quad \boxed{+2}$$

$$6 \quad d. \tan \frac{x}{2} = \frac{1 - \cos x}{\sin x} \quad \boxed{+2}$$

$$= \frac{1 + \frac{3}{7}}{\frac{\sqrt{40}}{7}} = \frac{10}{7} \cdot \frac{7}{\sqrt{40}} = \frac{10}{\sqrt{40}} \quad \boxed{+2}$$

$$\boxed{+2} \quad = \frac{10}{2\sqrt{10}} = \frac{\sqrt{10}}{2}$$

4. (24 points) Let  $\mathbf{u} = \langle -1, 3 \rangle$ ,  $\mathbf{v} = \langle 2, 2 \rangle$ . Find each of the following:

3 a.  $4\mathbf{u} - 2\mathbf{v}$

$$= \langle -4, 12 \rangle - \langle 4, 4 \rangle = \langle -8, 8 \rangle \quad \boxed{+2 \text{ for single error}}$$

3 b.  $\mathbf{u} \cdot \mathbf{v} = (-1)(2) + 3 \cdot 2 = -2 + 6 = 4 \quad \boxed{+2}$

6 c.  $\text{proj}_{\mathbf{v}} \mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2} \cdot \mathbf{v} = \frac{4}{2^2 + 2^2} \cdot \mathbf{v} = \frac{1}{2} \vec{v} = \langle 1, 1 \rangle \quad \boxed{+2}$

6 d.  $(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) = \underbrace{\langle 1, 5 \rangle}_{+2} \cdot \underbrace{\langle -3, 1 \rangle}_{+2} = -3 + 5 = 2 \quad \boxed{+2}$

6 e. Resolve  $\mathbf{u}$  into  $\mathbf{u}_1$  and  $\mathbf{u}_2$  such that  $\mathbf{u}_1$  is parallel to  $\mathbf{v}$  and  $\mathbf{u}_2$  is perpendicular to  $\mathbf{v}$ .

$$\mathbf{u}_1 = \text{proj}_{\vec{v}} \vec{u} = \langle 1, 1 \rangle \quad \boxed{+2}$$

$$\mathbf{u}_2 = \mathbf{u} - \mathbf{u}_1 = \underbrace{\langle -1, 3 \rangle}_{+2} - \langle 1, 1 \rangle = \langle -2, 2 \rangle \quad \boxed{+2}$$

5. (16 points) A jet is flying through a wind which is blowing 60 mph in direction due east. The jet has a speed of 620 mph relative to air and is headed in the direction of N 45° E. Find the true speed and direction of the jet.

Let  $\vec{u}$  be plane vector,  $\vec{v}$  be wind

rewrite  $\vec{u}$  and  $\vec{v}$

$$\begin{aligned}\vec{u} &= 620 \cos 45^\circ i + 620 \sin 45^\circ j \\ &= 620 \frac{\sqrt{2}}{2} i + 620 \frac{\sqrt{2}}{2} j \\ &= 310\sqrt{2} i + 310\sqrt{2} j \quad ] + 4\end{aligned}$$

$$\begin{aligned}\vec{v} &= 60 \cos 0^\circ i + 60 \sin 0^\circ j \\ &= 60i \quad = 60i \quad ] + 4\end{aligned}$$

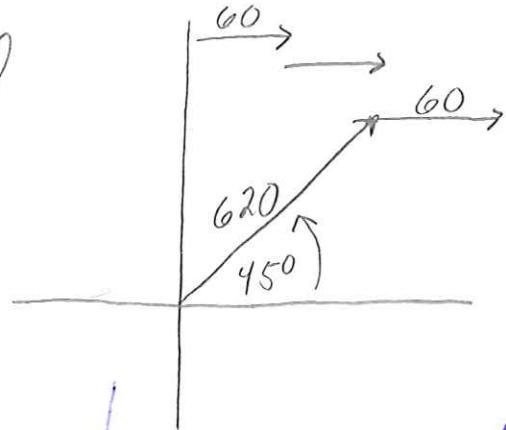
$$\text{True velocity} = \vec{u} + \vec{v} = (310\sqrt{2} + 60)\vec{i} + 310\sqrt{2}\vec{j}$$

$$\text{True speed} = |\vec{u} + \vec{v}| = \sqrt{(310\sqrt{2} + 60)^2 + (310\sqrt{2})^2} \quad ] + 4$$

$$\approx 664 \text{ mph}$$

$$\text{Direction: } \theta = \tan^{-1} \left( \frac{310\sqrt{2}}{310\sqrt{2} + 60} \right) \approx 41.34^\circ$$

+ 4



not required, but  
worth 4 pts if  
correct w/ labels