

Eastern Oregon University  
Concurrent Enrollment/Credit by Proficiency Program

Math 112, Spring, 2015

Final Exam

name/school: Key

Show any relevant work. For each problem, circle your answer.

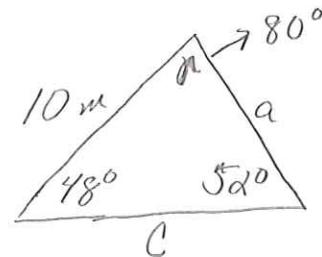
1. (24 points) Solve each triangle below. Part of your task is to determine whether to use the Law of Sines or the Law of Cosines.

1Q a.  $\alpha = 48^\circ$ ,  $\beta = 52^\circ$ ,  $b = 10$  meters

$$\gamma = 180^\circ - (48^\circ + 52^\circ) = 80^\circ \boxed{+3}$$

$$\frac{\sin 48^\circ}{a} = \frac{\sin 52^\circ}{10} \Rightarrow a = \frac{10 \cdot \sin 48^\circ}{\sin 52^\circ} \approx 9.43 \text{ m}$$

+6  
~~10~~



$$\frac{\sin 80^\circ}{c} = \frac{\sin 52^\circ}{10} \Rightarrow c = \frac{10 \cdot \sin 80^\circ}{\sin 52^\circ} \approx 12.5 \text{ m} \boxed{+3}$$

1Q b.  $a = 11$ ,  $b = 5$ ,  $c = 11$

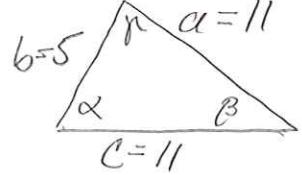
$$a^2 = b^2 + c^2 - 2bc \cdot \cos \alpha$$

$$11^2 = 5^2 + 11^2 - 2 \cdot 5 \cdot 11 \cdot \cos \alpha \boxed{+3}$$

$$-25 = -110 \cdot \cos \alpha$$

$$\alpha = \cos^{-1}\left(\frac{25}{110}\right) \approx 77^\circ, \quad \beta = 77^\circ \boxed{+3}$$

$$\beta = 180^\circ - 2(77^\circ) = 26^\circ \boxed{+3}$$



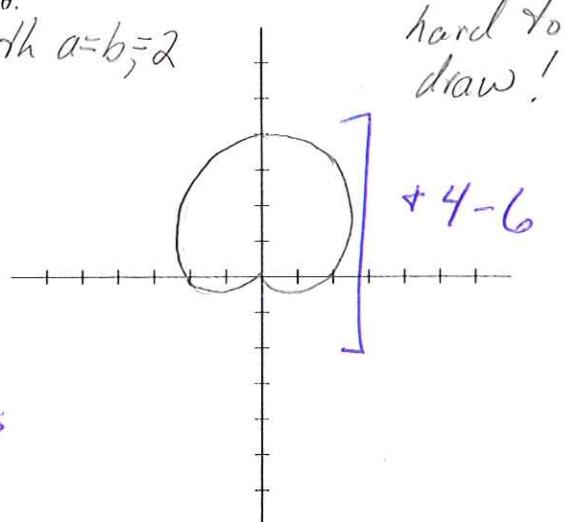
2. (10 points) Sketch the graph of the polar equation;  $r = 2 + 2 \sin \theta$ .

+3  
but not required  
This has the form  $r = a + b \sin \theta$ , with  $a = b = 2$   
the graph is a cardioid

$\theta$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$
$r$	2	$2 + 2\sqrt{2}$	4	2	$2 - 2\sqrt{2}$

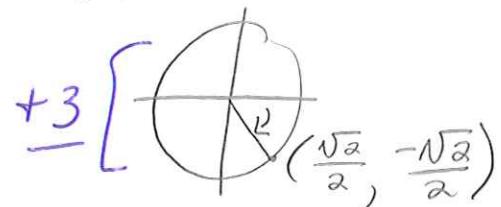
+3-4

or some indication of how values were found.

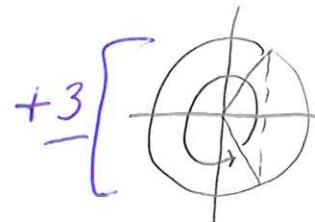


3. (24 points) For each of the following, construct any appropriate reference triangles, and find exact values.

6 a.  $\sin^{-1}(-\frac{\sqrt{2}}{2}) = -\frac{\pi}{4}$  +3



6 b.  $\cos^{-1}(\cos(5\pi/3)) = \cos^{-1}(\cos \frac{\pi}{3}) = \frac{\pi}{3}$  +3



12 c. If  $\sin x = -1/3$  and  $\cos x > 0$ , find the values of the other five elementary trigonometric functions at  $x$ .

$$\cos x = \frac{2\sqrt{2}}{3}$$

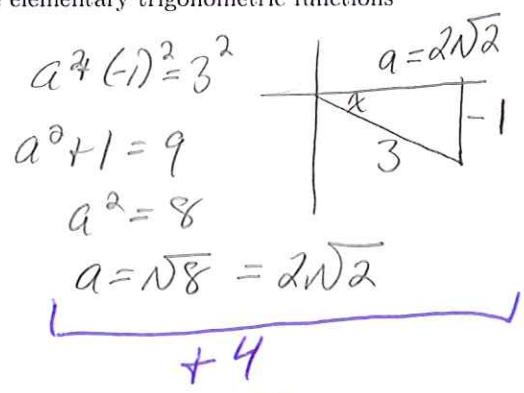
$$\tan x = \frac{-1}{2\sqrt{2}}$$

$$\sec x = \frac{3}{2\sqrt{2}}$$

$$\cot x = -2\sqrt{2}$$

$$\csc x = -3$$

+8



4. (10 points) Find all cube roots of 1, and graph the roots in the complex plane.

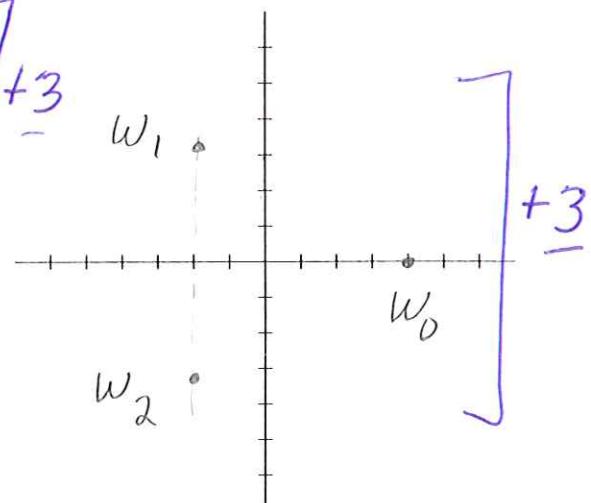
$$z^3 = 1 + 0i = 1 \cdot (\cos 0 + i \sin 0)$$

$r=1$ ,  $1^{1/3}=1$ , three roots

$$w_0 = 1[\cos 0 + i \sin 0] = 1$$

$$w_1 = 1[\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}]$$

$$w_2 = 1[\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}]$$



5. (10 points) Given the point with polar coordinates  $(-2, \pi/3)$ , convert to rectangular coordinates.

$$x = r\cos\theta = -2\cos\left(\frac{\pi}{3}\right) = -2\left(\frac{1}{2}\right) = -1 \quad \boxed{+5}$$

$$y = r\sin\theta = -2\sin\left(\frac{\pi}{3}\right) = -2\left(\frac{\sqrt{3}}{2}\right) = -\sqrt{3} \quad \boxed{+5}$$

6. (12 points) For the following function, sketch one period of the graph carefully, and label the grid sufficiently to indicate the period and either amplitude or asymptotes.

$$f(x) = 2\cos(x - \frac{\pi}{2}) - 2$$

$$\text{amplitude} = 2 \quad \boxed{+2}$$

$$\text{period} = 2\pi \quad \boxed{+2}$$

$$\text{shift right by } \frac{\pi}{2} \quad \boxed{+2}$$

$$\text{shift down by } 2 \quad \boxed{+2}$$

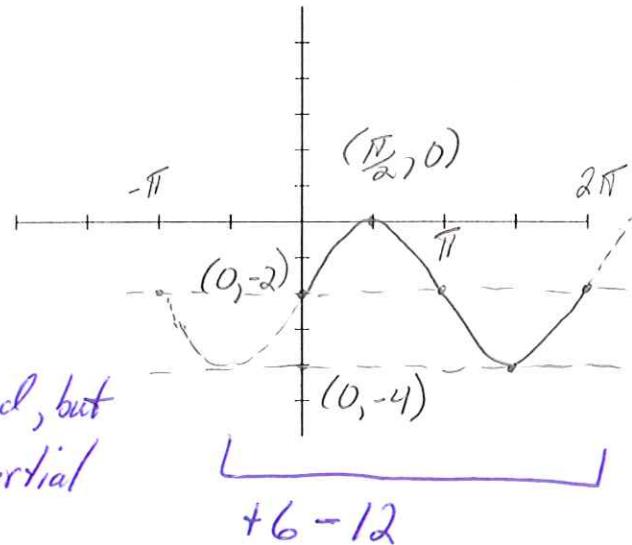
$$f(0) = 2\cos(-\frac{\pi}{2}) - 2 = -2$$

$$f\left(\frac{\pi}{2}\right) = 2\cos(0) - 2 = 0$$

$$f(\pi) = 2\cos(\pi) - 2 = -2$$

$$f\left(\frac{3\pi}{2}\right) = 2\cos\pi - 2 = -4$$

- need only be indicated on graph



7. (10 points) Let  $\mathbf{u} = \langle 3, -1 \rangle$ ,  $\mathbf{v} = \langle 6, 3 \rangle$ .

Find  $\text{proj}_{\mathbf{v}} \mathbf{u}$  and resolve  $\mathbf{u}$  into  $\mathbf{u}_1$  and  $\mathbf{u}_2$  such that  $\mathbf{u}_1$  is parallel to  $\mathbf{v}$  and  $\mathbf{u}_2$  is perpendicular to  $\mathbf{v}$ .

$$\text{proj}_{\mathbf{v}} \mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \mathbf{v} = \frac{18 - 3}{45} \langle 6, 3 \rangle = \frac{1}{3} \langle 6, 3 \rangle = \langle 2, 1 \rangle = \mathbf{u}_1 \quad \boxed{+3}$$

$$\mathbf{u} \cdot \mathbf{v} = 18 - 3 = 15 \quad \boxed{+2}$$

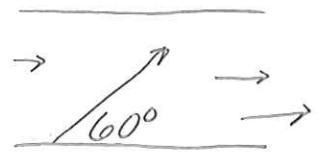
$$\|\mathbf{v}\|^2 = 6^2 + 3^2 = 45 \quad \boxed{+2}$$

$$\begin{aligned} \mathbf{u}_2 &= \langle 3, -1 \rangle - \langle 2, 1 \rangle \\ &= \langle 1, -2 \rangle \end{aligned} \quad \boxed{+3}$$

8. (20 points) A straight river flows east at a speed of 6 mi/hr. A kayaker leaves the south shore of the river on a bearing of N 30° E (an angle 60° to the river bank.) The kayaker has a speed of 4 mi/h relative to the water.

a. Express the velocity of the river as a vector in component form.

$$\begin{aligned} \cancel{+4} \\ \vec{V} &= 6 \cos 0^\circ \hat{i} + 6 \sin 0^\circ \hat{j} \\ &= 6\hat{i}, \text{ or } \langle 6, 0 \rangle \end{aligned}$$



b. Express the velocity of the kayaker relative to the water as a vector in component form.

$$\begin{aligned} \cancel{+4} \\ \vec{a} &= 4 \cos 60^\circ \hat{i} + 4 \sin 60^\circ \hat{j} \\ &= 4\left(\frac{1}{2}\right)\hat{i} + 4\left(\frac{\sqrt{3}}{2}\right)\hat{j} = 2\hat{i} + 2\sqrt{3}\hat{j} \\ &\text{or } \langle 2, 2\sqrt{3} \rangle \end{aligned}$$

c. Find the true velocity of the kayaker.

$$\begin{aligned} \cancel{+4} \\ \vec{w} &= \vec{a} + \vec{V} = (2 + 6)\hat{i} + 2\sqrt{3}\hat{j} \\ &= 8\hat{i} + 2\sqrt{3}\hat{j}, \text{ or } \langle 8, 2\sqrt{3} \rangle \end{aligned}$$

d. Find the true speed and direction of the kayaker.

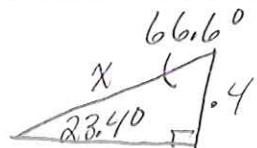
$$\text{Speed} = |w| = \sqrt{8^2 + (2\sqrt{3})^2} = \sqrt{64 + 12} \approx 8.72 \text{ mi/hr} \quad \cancel{+4}$$

$$\begin{aligned} \text{Direction: } \theta &= \tan^{-1}\left(\frac{2\sqrt{3}}{8}\right) \approx 23.4^\circ \text{ to river bank, or} \\ &N 66.6^\circ E \quad \cancel{+4} \end{aligned}$$

Bonus! If the river is .04 miles wide along this straight stretch, how far has the kayaker traveled when she reaches the far side?

$$\frac{5 \text{ mi } 90^\circ}{x} = \frac{5 \text{ mi } 23.4^\circ}{.04}$$

$$x = \frac{.04(1)}{\sin 23.4^\circ} \approx .1 \text{ mi.}$$



+5  
all or  
nothing