

Eastern Oregon University
Concurrent Enrollment/Credit by Proficiency Program

Math 112, Spring, 2014

Final Exam

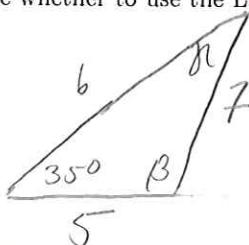
name/school: Key

Show any relevant work. For each problem, circle your answer

1. (20 points) Solve each triangle below. Part of your task is to determine whether to use the Law of Sines or the Law of Cosines.

10 a. $\alpha = 35^\circ$, $c = 5$ meters, $a = 7$ meters

$$\frac{\sin 35^\circ}{7} = \frac{\sin \gamma}{5} = \frac{\sin \beta}{b}$$



$$\gamma = \sin^{-1}\left(\frac{5 \cdot \sin 35^\circ}{7}\right) \approx 24.2^\circ \quad]+5$$

$$\beta = 180^\circ - (35^\circ + 24.2^\circ) = 120.8^\circ \quad]+2$$

10 b. $a = 8$, $b = 8$, $c = 14$

$$14^2 = 8^2 + 8^2 - 2 \cdot 8 \cdot 8 \cdot \cos \gamma \quad]+5$$

$$\cos \gamma = \frac{14^2 - 128}{-128}$$

$$\gamma = \cos^{-1}\left(\frac{14^2 - 128}{-128}\right) \approx 122^\circ$$

$$\begin{aligned} b &= \frac{7 \cdot \sin 120.8^\circ}{\sin 35^\circ} \\ &= 10.48 \text{ meters} \end{aligned} \quad]+3$$



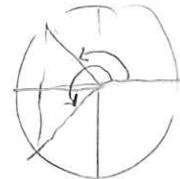
2. (20 points) Solve each of the following equations:

10 a. $2 \cos x + \sqrt{3} = 0$, $0 \leq x \leq 2\pi$

$$\cos x = \frac{-\sqrt{3}}{2} \quad]+4$$

$$x = \frac{5\pi}{6}, \frac{7\pi}{6} \quad]+3 \quad]+3$$

$$\alpha - \beta = \frac{(180 - 122)^\circ}{2} \approx 29^\circ \quad]+3$$



10 b. $2 \sin^2 x - \sin x = 0$, all real x .

$$\sin x (2 \sin x - 1) = 0 \quad]+2$$

$$\sin x = 0 \quad \text{or} \quad \sin x = \frac{1}{2} \quad]+2$$

$$0 + \pi k$$

$$\frac{\pi}{6} + 2\pi k$$

$$\frac{5\pi}{6} + 2\pi k$$

$$x = \begin{cases} 0 + \pi k \\ \frac{\pi}{6} + 2\pi k \\ \frac{5\pi}{6} + 2\pi k \end{cases}, \quad k \text{ is any integer}$$

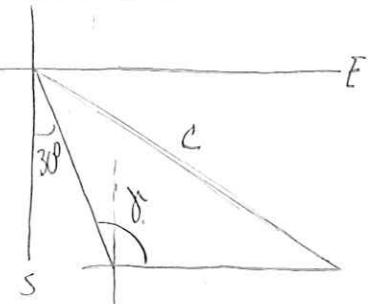
3. (20 points) An airplane leaves PDX on a compass heading of S 30° E with an airspeed of 420 miles per hour. After 90 minutes, the pilot changes to a bearing of 90° , or due east, and continues on this leg for another 90 minutes. How far is the plane from PDX at this time?

$$1.5 \text{ hrs} \times 420 \text{ mph} = 630 \text{ mi, each leg } \boxed{+4}$$

$$\text{angle } \gamma = 120^\circ \boxed{+5}$$

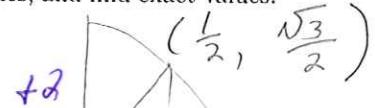
$$c^2 = [630^2 + 630^2 - 2 \cdot 630 \cdot 630 \cdot \cos 120^\circ] \boxed{+5}$$

$$c = \sqrt{[\quad]} \approx \underbrace{1091}_{+3} \underbrace{\text{miles}}_{+3}$$



4. (20 points) For each of the following, construct any appropriate reference triangles, and find exact values.

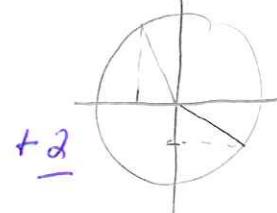
5 a. $\arctan(\sqrt{3})$ $\sqrt{3} = \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{3}/2}{1/2}$



$$\theta = \arctan \sqrt{3} = \frac{\pi}{3} \boxed{+3}$$

others possible

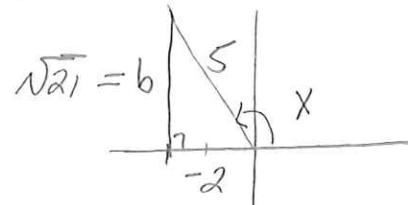
5 b. $\sin^{-1}(\cos(2\pi/3))$ $\cos \frac{2\pi}{3} = -\frac{1}{2}$



$$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6} \boxed{+3}$$

- 10 c. If $\cos x = -2/5$ and $\sin x > 0$, find the values of the other five elementary trigonometric functions at x .

$$b^2 = 5^2 - (-2)^2 = 25 - 4 = 21 \boxed{+4}$$



$$\sin x = \frac{\sqrt{21}}{5}$$

$$\sec x = -\frac{5}{2}$$

$$\tan x = -\frac{\sqrt{21}}{2}$$

$$\csc x = \frac{5}{\sqrt{21}}$$

$$\cot x = -\frac{2}{\sqrt{21}}$$

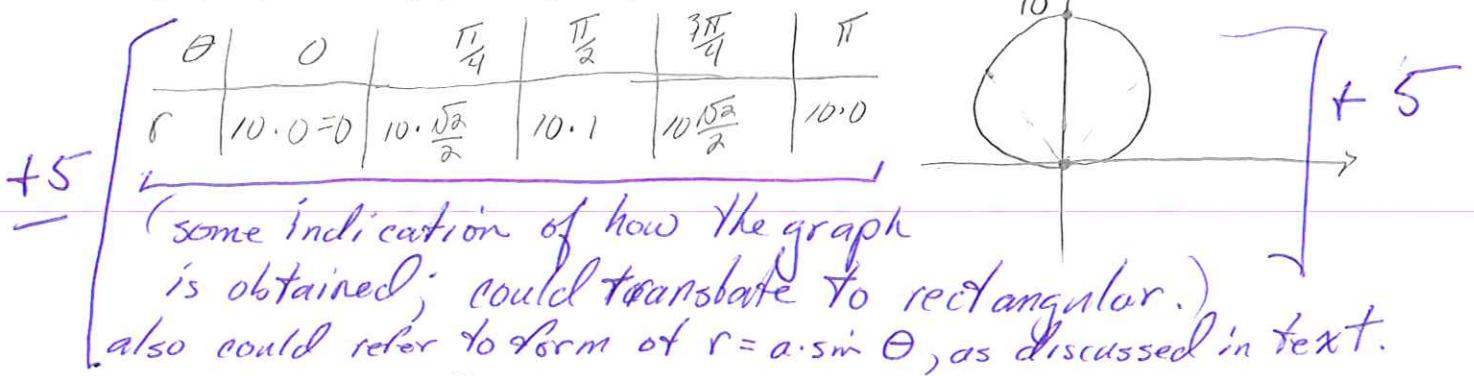
+6

5. (10 points) Given the point with polar coordinates $(3, \pi/3)$, convert to rectangular coordinates with three decimal places of precision.

$$x = r \cdot \cos \theta = 3 \cdot \cos \frac{\pi}{3} = 3 \left(\frac{1}{2} \right) = \frac{3}{2} \quad] + 5$$

$$y = r \cdot \sin \theta = 3 \cdot \sin \frac{\pi}{3} = 3 \left(\frac{\sqrt{3}}{2} \right) = \frac{3\sqrt{3}}{2} \quad] + 5$$

6. (10 points) Sketch the graph of the polar equation; $r = 10 \sin \theta$.



7. (10 points) Let $z_1 = -1 + \sqrt{3}i$, $z_2 = 1 + i$. Write z_1 and z_2 in polar form and find each of the following:

a. $z_1 z_2$

$$z_1 = 2 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

$$z_2 = \sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

]

+ 4

b. z_1/z_2

$$z_1: r = \sqrt{1+3} = 2$$

$$\theta = \tan^{-1}(-\sqrt{3})$$

$$= \frac{2\pi}{3}$$

$$a) z_1 z_2 = 2\sqrt{2} \left(\cos \left(\frac{2\pi}{3} + \frac{\pi}{4} \right) + i \sin \left(\frac{2\pi}{3} + \frac{\pi}{4} \right) \right) \quad] + 3$$

$$z_2: r = \sqrt{1+1} = \sqrt{2}$$

$$b) \frac{z_1}{z_2} = \frac{2}{\sqrt{2}} \left(\cos \left(\frac{2\pi}{3} - \frac{\pi}{4} \right) + i \sin \left(\frac{2\pi}{3} - \frac{\pi}{4} \right) \right) \quad] + 3$$

$$\theta = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$$

8. (10 points) Find all cube roots of 1, and graph the roots in the complex plane.

$$z = 1 + 0i = 1 \left(\cos 0 + i \sin 0 \right)$$

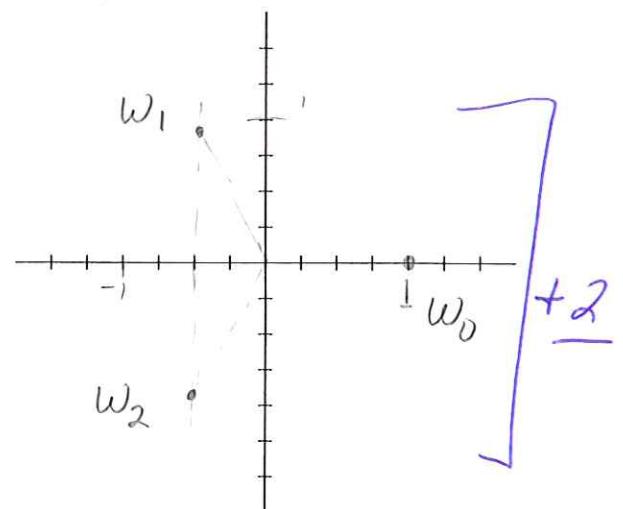
$r = 1$, $1^{\frac{1}{3}} = 1$, three roots

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+ 3

$$w_0 = 1 \left[\cos 0 + i \sin 0 \right]$$



$$w_1 = 1 \left[\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right]$$

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+ 5

$$w_2 = 1 \left[\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right]$$

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