Eastern Oregon University
Concurrent Enrollment/Credit by Proficiency Program

Math 112, Spring, 2014 Exam 3 name/school: Key

Show any relevant work. For each problem, circle your answer

---

1. (20 points) Verify each of the following identities:

10 a. $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

$(\sin x + \cos x)^2 = \sin^2 x + 2\sin x \cdot \cos x + \cos^2 x \,]\, +3$

$= (\sin^2 x + \cos^2 x) + 2\sin x \cdot \cos x \,]\, +3$

$= 1 + 2\sin x \cdot \cos x \,]\, +4$

10 b. $\cos(x - \frac{\pi}{2}) = \sin x$ (Use a sum or difference formula.)

$\cos(x - \frac{\pi}{2}) = \cos x \cdot \cos \frac{\pi}{2} + \sin x \cdot \sin \frac{\pi}{2} \,]\, +4$

$= \cos x \cdot 0 + \sin x \cdot 1 \,]\, +4$

$= \sin x \,]\, +2$

2. (20 points) Use addition or subtraction formulas to evaluate each of the following expressions.

10 a. $\sin \frac{11\pi}{12} = \sin\left(\frac{8\pi}{12} + \frac{3\pi}{12}\right) = \sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) \,]\, +4$

$+3 \,[\, = \sin \frac{2\pi}{3} \cdot \cos \frac{\pi}{4} + \cos \frac{2\pi}{3} \cdot \sin \frac{\pi}{4}$

$+3 \,[\, = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \left(-\frac{1}{2}\right)\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}-1}{2}\right)$

+3

10 b. $\cos \frac{3\pi}{7} \cos \frac{2\pi}{21} + \sin \frac{3\pi}{7} \sin \frac{2\pi}{21} = \cos \frac{9\pi}{21} \cdot \cos \frac{2\pi}{21} + \sin \frac{9\pi}{21} \cdot \sin \frac{2\pi}{21}$

$= \cos\left(\frac{9\pi - 2\pi}{21}\right) = \cos \frac{7\pi}{21} = \cos \frac{\pi}{3} \,]\, +2$

+3

$= \frac{1}{2} \,]\, +2$

3. (16 points) Use double-angle or half-angle formulas to evaluate each of the following expressions.

8 a. $\cos 2x$, if $\sin x = -\frac{3}{7}$ and x is a quadrant III angle.

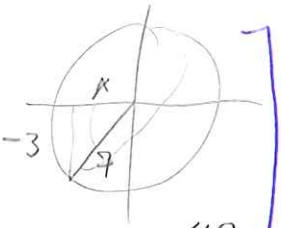
$\cos 2x = \cos^2 x - \sin^2 x$

$= \frac{40}{49} - \frac{9}{49} = \frac{31}{49}$ +4

$x^2 + 9 = 49$

$x^2 = 40 \quad x = 2\sqrt{10}$

$\cos x = \frac{\sqrt{40}}{7}, \quad \cos^2 x = \frac{40}{49}$



+4

or

or $\cos 2x = 2\cdot\cos^2 x - 1$

$= \frac{80}{49} - \frac{49}{49} = \frac{31}{49}$ +4

8 b. $\cos 15°$

$= \cos\left(\frac{30}{2}\right) = \pm\sqrt{\frac{1+\cos 30}{2}}$ +2

quadrant 1

$= (+)\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$ +4

+2

simplification not required but must be correct if done.

4. (16 points) Find all solutions to each equation:

8 a. $\sin^2\theta - \sin\theta - 2 = 0$

$\sin^2\theta - \sin\theta - 2 = (\sin\theta - 2)(\sin\theta + 1) = 0$ +2

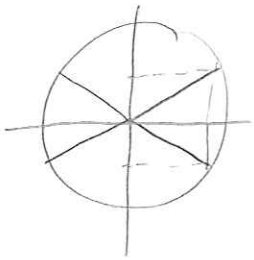
if $\sin\theta = 2$ or $\sin\theta = -1$ +2

no solutions

+2

$\sin\theta = -1$ if $\theta = -\frac{\pi}{2} + 2\pi k$ for any integer k. +2

8 b. $3\tan^2\theta - 1 = 0, \quad 0 \le \theta \le 2\pi$



$3\tan^2\theta = 1$

$\tan^2\theta = \frac{1}{3}$ +2

$\tan\theta = \pm\frac{1}{\sqrt{3}}$ +2

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \text{ or } \frac{11\pi}{6}$

+4

5. (20 points) Let $\mathbf{u} = \langle -2, 5\rangle$, $\mathbf{v} = \langle 3, 3\rangle$, $\mathbf{w} = \langle 7, 1\rangle$. Find each of the following:

5 a. $4\mathbf{u} - 2\mathbf{v}$

$$\underbrace{\langle -8, 20\rangle - \langle 6, 6\rangle}_{+3} = \underbrace{\langle -14, 14\rangle}_{+2}$$

(not required)

5 b. $\mathbf{u} + 2\mathbf{v} - 2\mathbf{w}$

$$\underbrace{\langle -2, 5\rangle + \langle 6, 6\rangle - \langle 14, 2\rangle}_{+3} = \underbrace{\langle -10, 9\rangle}_{+2}$$

5 c. $|\mathbf{w}|$

$$= \underbrace{\sqrt{7^2 + 1^2}}_{+2} = \underbrace{\sqrt{50}}_{+3} \Big] + \_$$

5 d. $\left|\frac{1}{|\mathbf{w}|}\mathbf{w}\right|$.

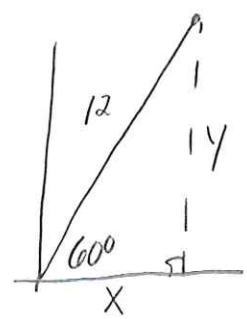
$$= \underbrace{\left|\frac{1}{\sqrt{50}}\langle 7, 1\rangle\right|}_{+2} = \left|\left\langle \frac{7}{\sqrt{50}}, \frac{1}{\sqrt{50}}\right\rangle\right|$$

$$= \sqrt{\frac{7^2}{50} + \frac{1^2}{50}} = \sqrt{\frac{50}{50}} = 1 \Big] + 3$$

6. (8 points) Find the horizontal and vertical components of the vector $\mathbf{v}$ with the given length and direction, and rewrite $\mathbf{v}$ in terms of the vectors $\mathbf{i}$ and $\mathbf{j}$: $|\mathbf{v}| = 12$, $\theta = 60°$

$$Y = 12 \cdot \sin 60° = 12 \cdot \frac{\sqrt{3}}{2} = 6\sqrt{3} \Big]$$

$$X = 12 \cdot \cos 60° = 12\left(\tfrac{1}{2}\right) = 6$$



not required

$$\vec{V} = \underbrace{6\vec{i}}_{+4} + \underbrace{6\sqrt{3}\,\vec{j}}_{+4}$$