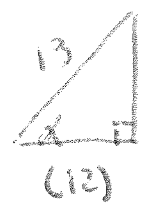
Name ______________________________ Date ______________

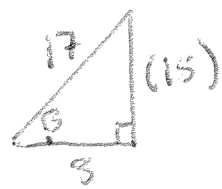
Math 112: #38 A/B/C/D

A) Given that $\sin\alpha = \frac{5}{13}$ and $\cos\beta = \frac{8}{17}$ and that α, β each lie in the first quadrant, use the sum identities: $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ and $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$



$\cos\alpha = \frac{12}{13}$

$\tan\alpha = 5/12$



$\sin\beta = \frac{15}{17}$

$\tan\beta = \frac{15}{8}$

to find the exact values of:

1. $\sin(\alpha+\beta)$ $= \frac{5}{13}\cdot\frac{8}{17} + \frac{12}{13}\cdot\frac{15}{17}$

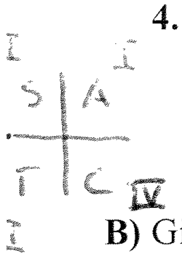
   $\frac{40}{221} + \frac{180}{221} = \boxed{\frac{220}{221}}$

2. $\cos(\alpha+\beta)$ $= \frac{12}{13}\cdot\frac{8}{17} - \frac{5}{13}\cdot\frac{15}{17} = \frac{96}{221} - \frac{75}{221} = \boxed{\frac{21}{221}}$

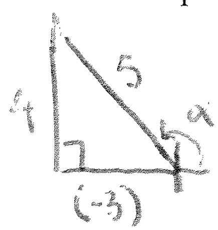
3. $\tan(\alpha+\beta)$, and $= \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{220/221}{21/221} = \boxed{\frac{220}{21}}$

4. Determine the quadrant in which α + β lies.

   $\sin(\alpha+\beta) > 0$ and $\cos(\alpha+\beta) > 0$ so $\alpha+\beta$ is in I

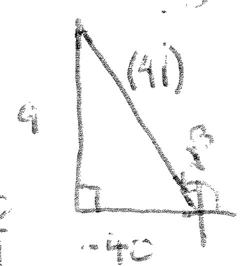


B) Given that $\sin\alpha = \frac{4}{5}$ and $\tan\beta = -\frac{9}{40}$ and that α, β each lie in the second quadrant, use the sum identities: $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ and $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$



$\cos\alpha = \frac{-3}{5}$

$\tan\alpha = -\frac{4}{3}$



$\sin\beta = \frac{9}{41}$

$\cos\beta = -\frac{40}{41}$

to find the exact values of:

1. $\sin(\alpha+\beta)$ $= \frac{4}{5}\cdot\frac{-40}{41} + \frac{-3}{5}\cdot\frac{9}{41}$

   $= \frac{-160}{205} + \frac{-27}{205} = \boxed{-\frac{187}{205}}$

2. $\cos(\alpha+\beta)$ $= -\frac{3}{5}\cdot\frac{-40}{41} - \frac{4}{5}\cdot\frac{9}{41}$

   $\frac{120}{205} - \frac{36}{205} = \boxed{\frac{84}{205}}$

3. $\tan(\alpha+\beta)$, and

   $= \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{-187/\cancel{205}}{84/\cancel{205}} = \boxed{-\frac{187}{84}}$

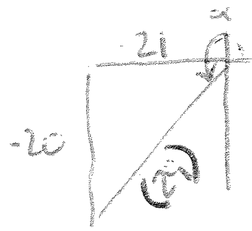
4. Determine the quadrant in which α + β lies.

   $\sin(\alpha+\beta) < 0$

   $\cos(\alpha+\beta) > 0$ so $\alpha+\beta$ is in IV

C) Given that $\tan\alpha = \frac{20}{21}$ and $\sin\beta = -\frac{3}{5}$ and that α, β each lie in the third quadrant, use the sum identities: sin(α + β) = sin α cos β + cos α sin β
and cos(α + β) = cos α cos β − sin α sin β

to find the exact values of:



$\sin\alpha = \frac{-20}{29}$

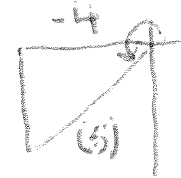
$\cos\alpha = \frac{-21}{29}$

1. sin(α + β) $= \frac{-20}{29}\cdot\frac{-4}{5} + \frac{-21}{29}\cdot\frac{-3}{5}$

$\frac{80}{145} + \frac{63}{145} = \boxed{\frac{143}{145}}$

2. cos(α + β)

$\cos\beta = \frac{-4}{5}$



$= \frac{-21}{29}\cdot\frac{-4}{5} - \frac{-20}{29}\cdot\frac{-3}{5} = \frac{84}{145} - \frac{60}{145} = \boxed{\frac{24}{145}}$

3. tan(α + β), and

$= \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{143/145}{24/145} = \boxed{\frac{143}{24}}$

4. Determine the quadrant in which α + β lies.

$\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$ & $\tan(\alpha+\beta) > 0$ $\boxed{\text{therefore Quadrant I}}$

D) Given that $\tan\alpha = -\frac{5}{12}$ and $\cos\beta = \frac{12}{37}$ and that α, β each lie in the fourth quadrant, use the sum identities: sin(α + β) = sin α cos β + cos α sin β
and cos(α + β) = cos α cos β − sin α sin β

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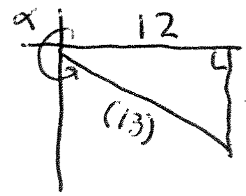
to find the exact values of:



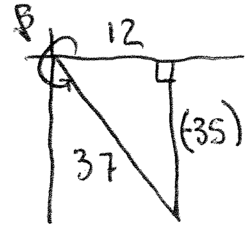
$\sin\alpha = \frac{-5}{13}$

$\cos\alpha = \frac{12}{13}$

1. sin(α + β) $= \frac{-5}{13}\cdot\frac{12}{37} + \frac{12}{13}\cdot\frac{-35}{37}$

$\frac{-60}{481} + \frac{-420}{481} = \boxed{\frac{-480}{481}}$

$\sin\beta = \frac{-35}{37}$



2. cos(α + β) $\frac{12}{13}\cdot\frac{12}{37} - \frac{-5}{13}\cdot\frac{-35}{37} = \frac{144}{481} - \frac{175}{481} = \boxed{\frac{-31}{481}}$

3. tan(α + β), and

$= \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{-480/481}{-31/481} = \boxed{\frac{480}{31}}$

4. Determine the quadrant in which α + β lies.

$\cos, \sin(\alpha+\beta) < 0$

$\tan(\alpha+\beta) > 0$

$\boxed{\text{III Quadrant}}$