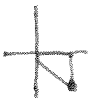
Name ______________________________ Date ______________

Math 112: #36 A/B/C

A) Suppose $z = 4\sqrt{3} - 4i$ $\quad r = 8 \quad \theta = -\frac{\pi}{6} = \frac{11\pi}{6} \quad w = 8\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$



1. Find z^5.

$$w^5 = 8^5\left(\cos 5\cdot\frac{11\pi}{6} + i\sin 5\cdot\frac{11\pi}{6}\right)$$
$$= 8^5\left(\cos\frac{55\pi}{6} + i\sin\frac{55\pi}{6}\right)$$
$$= 32768\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$$
$$\boxed{= -16384\sqrt{3} - 16384i}$$

2. Find all complex numbers w satisfying $w = \sqrt[5]{z}$. $\quad n = 5,\ k = 0, 1, 2, 3, 4$

$$w_0 = 8^{1/5}\left(\cos\frac{11\pi}{6}\cdot\frac{1}{5} + i\sin\frac{11\pi}{6}\cdot\frac{1}{5}\right) = \boxed{8^{1/5}\left(\cos\frac{11\pi}{30} + i\sin\frac{11\pi}{30}\right)}$$

add $\frac{2\pi}{5} = \frac{12\pi}{30}$ each time

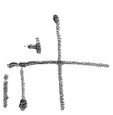
$$w_1 = 8^{1/5}\left(\cos\frac{11\pi}{30} + \frac{12\pi}{30} + i\sin\frac{11\pi}{30} + \frac{12\pi}{30}\right) = \boxed{8^{1/5}\left(\cos\frac{23\pi}{30} + i\sin\frac{23\pi}{30}\right)}$$

$$w_2 = 8^{1/5}\left(\cos\frac{35\pi}{30} + i\sin\frac{35\pi}{30}\right) \leftarrow \frac{7\pi}{6}$$
$$\boxed{= 8^{1/5}\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)}$$

$$w_3 = \boxed{8^{1/5}\left(\cos\frac{47\pi}{30} + i\sin\frac{47\pi}{30}\right)}$$

$$w_4 = \boxed{8^{1/5}\left(\cos\frac{59\pi}{30} + i\sin\frac{59\pi}{30}\right)}$$

B) Suppose $z = 4(-1 - i)$ $\quad r = 4\sqrt{2} \quad \theta = \frac{5\pi}{4} \quad w = 4\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$



1. Find z^4.

$$w^4 = (4\sqrt{2})^4\left(\cos\left(\cancel{4}\cdot\frac{5\pi}{\cancel{4}}\right) + i\sin\left(\cancel{4}\cdot\frac{5\pi}{\cancel{4}}\right)\right)$$
$$= 1024(\cos 5\pi + i\sin 5\pi)$$
$$= 1024(-1 + i\cdot 0)\ \boxed{= -1024}$$

2. Find all complex numbers w satisfying $w = \sqrt[3]{z}$. $\omega = 4\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

$n=3;\ k=0,1,2$

$$\omega_0 = (4\sqrt{2})^{1/3}\left(\cos\frac{5\pi}{3\cdot 4} + i\sin\frac{5\pi}{3\cdot 4}\right) = \boxed{(4\sqrt{3})^{1/3}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)}$$

add $\frac{2\pi}{3} = \frac{8\pi}{12}$

$$\omega_1 = (4\sqrt{2})^{1/3}\left(\cos\left(\frac{5\pi}{12}+\frac{8\pi}{12}\right) + i\sin\left(\frac{5\pi}{12}+\frac{8\pi}{12}\right)\right) = \boxed{(4\sqrt{3})^{1/3}\left(\cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12}\right)}$$

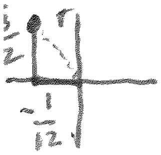
$$\omega_2 = (4\sqrt{2})^{1/3}\left(\cos\left(\frac{13\pi}{12}+\frac{8\pi}{12}\right) + i\sin\left(\frac{13\pi}{12}+\frac{8\pi}{12}\right)\right) = (4\sqrt{3})^{1/3}\left(\cos\frac{21\pi}{12} + i\sin\left(\frac{21\pi}{12}\right)\right)$$

$$= (4\sqrt{3})^{1/3}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

$$\boxed{= (4\sqrt{3})^{1/3}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)}$$

C) Suppose $z = \frac{1}{3}\left(-\frac{1}{4} + \frac{\sqrt{3}}{4}i\right)$

$$r = \sqrt{\left(-\frac{1}{12}\right)^2 + \left(\frac{\sqrt{3}}{12}\right)^2} = \sqrt{\frac{1}{144} + \frac{3}{144}} = \sqrt{\frac{4}{144}} = \frac{2}{12} = \frac{1}{6}$$



1. Find z^6.

$$\tan\theta = \frac{\sqrt{3}/12}{-1/12} = -\sqrt{3} = -\frac{\pi}{3} = \frac{2\pi}{3}$$

$$\omega = \frac{1}{6}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$\omega^6 = \left(\frac{1}{6}\right)^6\left(\cos\left(6\cdot\frac{2\pi}{3}\right) + i\sin\left(6\cdot\frac{2\pi}{3}\right)\right)$$

$$= \frac{1^6}{6^6}(\cos 4\pi + i\sin 4\pi) = \frac{1^6}{6^6}(1+0i) = \boxed{\frac{1^6}{6^6}}$$

2. Find all complex numbers w satisfying $w = \sqrt[4]{z}$. $n=4;\ k=0,1,2,3$

$$\omega_0 = \left(\frac{1}{6}\right)^{1/4}\left(\cos\left(\frac{1}{4}\cdot\frac{2\pi}{3}\right) + i\sin\left(\frac{1}{4}\cdot\frac{2\pi}{3}\right)\right) = \left(\frac{1}{6}\right)^{1/4}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

add $\frac{2\pi}{4} = \cancel{\frac{6\pi}{12}} = \frac{3\pi}{6}$ each time

$$\boxed{= (1/6)^{1/4}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)}$$

$$\omega_1 = \left(\frac{1}{6}\right)^{1/4}\left(\cos\left(\frac{\pi}{6}+\frac{3\pi}{6}\right) + i\sin\left(\frac{\pi}{6}+\frac{3\pi}{6}\right)\right) = (1/6)^{1/4}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$\boxed{= (1/6)^{1/4}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)}$$

$$\omega_2 = (1/6)^{1/4}\left(\cos\left(\frac{4\pi}{6}+\frac{3\pi}{6}\right) + i\sin\left(\frac{4\pi}{6}+\frac{3\pi}{6}\right)\right) = (1/6)^{1/4}\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$$

$$\boxed{= (1/6)^{1/4}\left(-\frac{\sqrt{3}}{2} + -\frac{1}{2}i\right)}$$

$$\omega_3 = (1/6)^{1/4}\left(\cos\left(\frac{7\pi}{6}+\frac{3\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}+\frac{3\pi}{6}\right)\right)$$

$$= (1/6)^{1/4}\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) \boxed{= (1/6)^{1/4}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)}$$