

Math 112: #33 A/B/C/D

A) Consider the polar equation $r^2 \cos 2\theta = 9$.

1. Explain why there can be no points of this graph on the y-axis.

Either $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ on the y-axis or $r^2 = r = 0$.
 since $\cos \frac{\pi}{2} = \cos \frac{3\pi}{2} = 0$, $r^2 \cos 2\theta = 0$ in both cases and $\neq 9$

2. Find a rectangular equation in the form: $x^2 - y^2 = r^2$ corresponding to $r^2 \cos 2\theta = 9$.

double angle

$$r^2 \cos 2\theta = 9 \rightarrow x, y$$

$$r^2 (\cos^2 \theta - \sin^2 \theta) = 9$$

$$-(1 - \cos^2 \theta)$$

$$r^2 (\cos^2 \theta - 1 + \cos^2 \theta) = 9$$

$$r^2 (2\cos^2 \theta - 1) = 9$$

$$r^2 (2\cos^2 \theta - 1) = 9$$

$$2r^2 \cos^2 \theta - r^2 = 9$$

$$2x^2 - (x^2 + y^2) = 9$$

$$x^2 - y^2 = 9$$

beautiful!

B) Consider the polar equation $r = \sqrt{1 + 2\cos 2\theta}$

1. Explain why there can be no points of this graph on the y-axis.

$\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ on the y-axis $\therefore 2\theta = \pi$ or 3π

$\cos \pi = \cos 3\pi = -1$ and $\sqrt{1 + 2(-1)} = \sqrt{-1} = i \rightarrow$ not in domain

However $\sqrt{1 + 2\cos(\frac{2\pi}{3})} = \sqrt{1 + 2\cos(\frac{4\pi}{3})} = 0$ so $(0, \frac{2\pi}{3}) \rightarrow 0, 0$
 and $(0, \frac{4\pi}{3}) \rightarrow 0, 0$
 are on the y-axis

2. Find a rectangular equation in the form: $x^2 - y^2 = r^2$ corresponding to $r = \sqrt{1 + 2\cos 2\theta}$

$$r = \sqrt{1 + 2\cos 2\theta} \rightarrow x, y$$

$$r^2 = 1 + 2\cos 2\theta$$

$$r^2 = 1 + 2(\cos^2 \theta - \sin^2 \theta)$$

$$r^2 = 1 + 2(2\cos^2 \theta - 1)$$

$$-2 - 1 \quad 1 \quad -2 \quad -$$

$$r^2 = 4\cos^2 \theta - 1$$

$$r^2 = 4\sin^2 \theta$$

$$r = 2\sin \theta$$

$$r^2 = 2r\sin \theta$$

$$x^2 + y^2 = 2x$$

complete the square

double angle

C) Consider the polar equation $r^2 \sin 2\theta = 9$.

1. Explain why there can be no points of this graph on the x-axis.

$\theta = 0$ or π , or $r = r^2 = 0$ to be on the x-axis
in both cases $r^2 \sin 2\theta = 0$ and $\neq 9$

2. Find a rectangular equation in the form: $y = f(x)$ corresponding to $r^2 \sin 2\theta = 9$.

$$r^2 \sin 2\theta = 9 \rightarrow x, y$$

Double
angle

$$r^2 \cdot 2 \sin \theta \cos \theta = 9$$

$$2 \cdot r \sin \theta \cdot r \cos \theta = 9$$

$$2 \cdot y \cdot x = 9$$

$$y = 9/2x$$

D) Consider the polar equation $r = 3 \sin 2\theta$.

1. Explain why there can be no points of this graph on the x-axis.

$\theta = 0$ or π , or $r = 0$ on the x-axis.

since $3 \sin(0) = 3 \sin(2\pi) = 0$, the graph will
go through $(0,0)$ on the x-axis

2. Find a rectangular equation in the form: $y = f(x)$ corresponding to $r = 3 \sin 2\theta$.

$$r = 3 \sin 2\theta \rightarrow x, y \quad x^2 + y^2 = r^2$$

Double
angle

$$r = 3 = 2 \sin \theta \cos \theta$$

$$r^2 = 6 r \sin \theta \cos \theta$$

$$r^4 = 6xy$$

$$(x^2 + y^2)^2 = 6xy$$

+ complete the square