Mine _____ Date _____

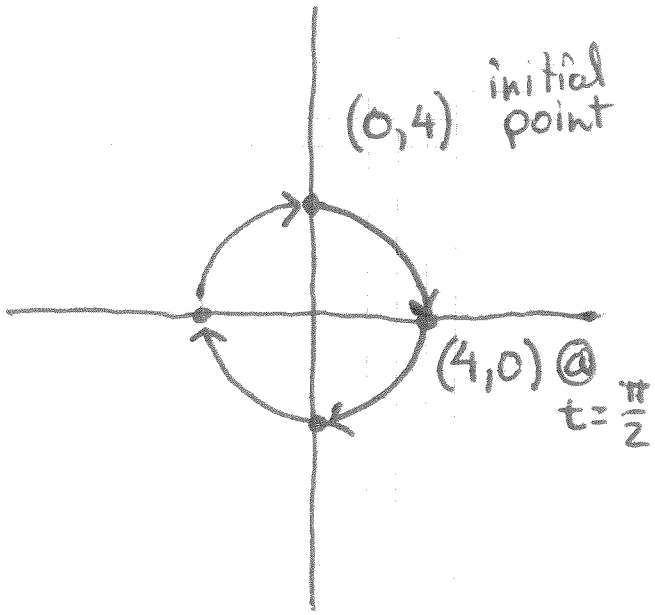
Math 112: #31 A/B/C/D

A) (No Calculator) Consider the parametric equations

$$x(t) = 4\sin t, \quad y(t) = 4\cos t, \quad 0 \le t \le 2\pi$$

1. Graph the curve in the xy-plane defined by these equations. Indicate on the graph three things: the initial point, the point corresponding to $t = \frac{\pi}{2}$, and the direction in which the curve is traversed as t goes from 0 to 2π.

t	x	y
0	0	4
$\pi/2$	4	0
π	0	-4
$\frac{3\pi}{2}$	-4	0
2π	0	4



2. Write a rectangular equation corresponding to the parametric equations above.

$\sin^2 t + \cos^2 t = 1$

$x^2 + y^2 = (4\sin t)^2 + (4\cos t)^2$

$= 16\sin^2 t + 16\cos^2 t$

$= 16(\sin^2 t + \cos^2 t)$ ($\sin^2 t + \cos^2 t = 1$)

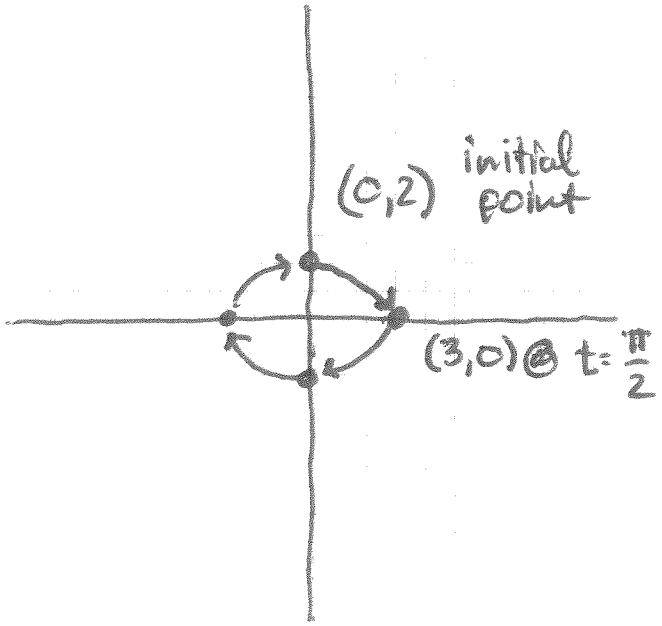
$x^2 + y^2 = 16$ circle w/ $r = 4$

BA) **(No Calculator)** Consider the parametric equations

$$x(t) = 3\sin t, \quad y(t) = 2\cos t, \quad 0 \le t \le 2\pi$$

1. Graph the curve in the xy-plane defined by these equations. Indicate on the graph three things: the initial point, the point corresponding to $t = \frac{\pi}{2}$, and the direction in which the curve is traversed as t goes from 0 to 2π.

t	x	y
0	0	2
$\pi/2$	3	0
π	0	-2
$\frac{3\pi}{2}$	-3	0
2π	0	2



2. Write a rectangular equation corresponding to the parametric equations above.

$x = 3\sin t$ $\qquad$ $y = 2\cos t$

$\frac{x}{3} = \sin t$ $\qquad$ $\frac{y}{2} = \cos t$

$\left(\frac{x}{3}\right)^2 = \sin^2 t$ $\qquad$ $\left(\frac{y}{2}\right)^2 = \cos^2 t$

$\sin^2 t + \cos^2 t = 1$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$ ok

$36\left(\frac{x^2}{9} + \frac{y^2}{4}\right) = 1 \cdot 36$ either

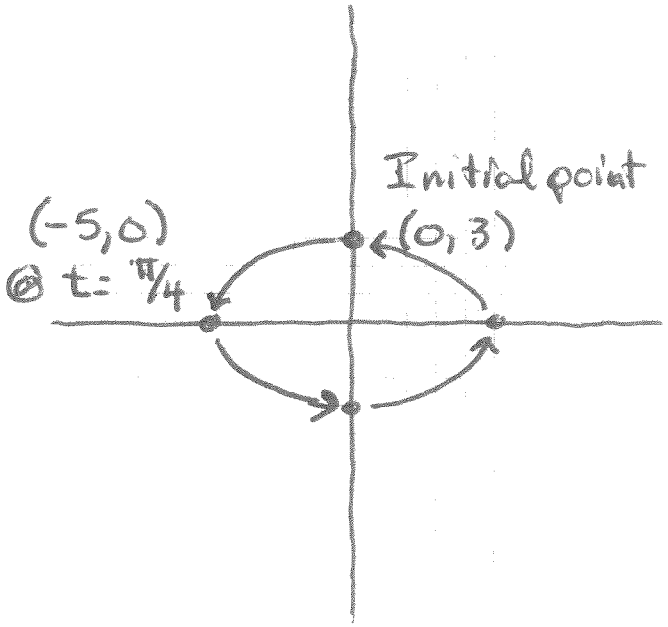
$4x^2 + 9y^2 = 36$ ok

C) **(No Calculator)** Consider the parametric equations

$$x(t) = -5\sin 2t, \quad y(t) = 3\cos 2t, \quad 0 \le t \le \pi$$

1. Graph the curve in the xy-plane defined by these equations. Indicate on the graph three things: the initial point, the point corresponding to $t = \frac{\pi}{4}$, and the direction in which the curve is traversed as t goes from 0 to π.

t	x	y
0	0	3
$\pi/4$	-5	0
$\pi/2$	0	-3
$\frac{3\pi}{4}$	5	0
π	0	3



2. Write a rectangular equation corresponding to the parametric equations above.

$x = -5\sin 2t$ $\quad y = 3\cos 2t$ $\quad$ let $\theta = 2t$

$\frac{x}{-5} = \sin\theta$ $\quad \frac{y}{3} = \cos\theta$

$\left(\frac{x}{-5}\right)^2 = \sin^2\theta$ $\quad \left(\frac{y}{3}\right)^2 = \cos^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{x}{-5}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$ ok

$225\left(\frac{x^2}{25} + \frac{y^2}{9}\right) = 1 \cdot 225$ either

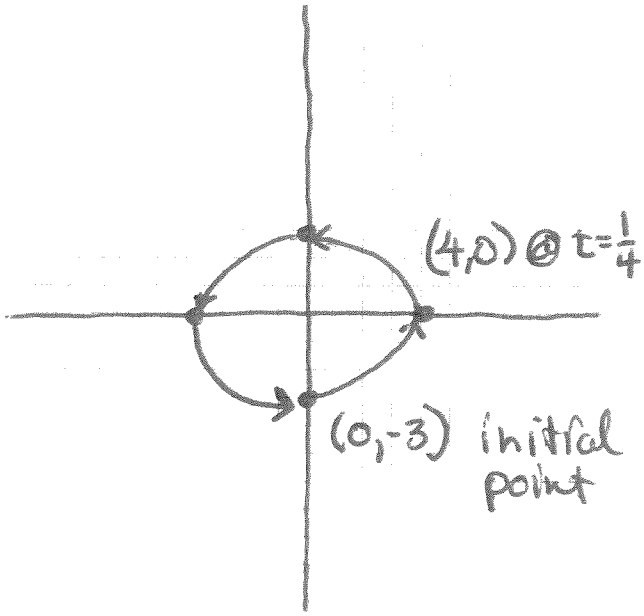
$9x^2 + 25y^2 = 225$ ok

D) (No Calculator) Consider the parametric equations

$$x(t) = 4\sin 2\pi t\,, \quad y(t) = -3\cos 2\pi t\,, \quad 0 \le t \le 1$$

1. Graph the curve in the xy-plane defined by these equations. Indicate on the graph three things: the initial point, the point corresponding to $t = \frac{1}{4}$, and the direction in which the curve is traversed as t goes from 0 to 1.

t	x	y
0	0	-3
1/4	4	0
1/2	0	3
3/4	-4	0
1	0	-3



2. Write a rectangular equation corresponding to the parametric equations above.

$\theta = 2\pi t$

$x = 4\sin\theta \qquad y = -3\cos\theta$

$\frac{x}{4} = \sin\theta \qquad \frac{y}{-3} = \cos\theta$

$\left(\frac{x}{4}\right)^2 = \sin^2\theta \qquad \left(\frac{y}{-3}\right)^2 = \cos^2\theta$

$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{-3}\right)^2 = \sin^2\theta + \cos^2\theta = 1$

ok $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{-3}\right)^2 = 1$

$144\left(\frac{x^2}{16} + \frac{y^2}{9} = 1\right)^{144}$ ok

$9x^2 + 16y^2 = 144$ ok