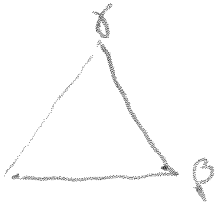


Math 112: #27 A/B/C/D

A) Solve the triangle specified by $\alpha = 51^\circ$, $a = 20$, $b = 21$.

$$h = 21 \sin 51^\circ = 16.32 \quad h < a < b$$

$$\Rightarrow 2 \text{ triangles}$$



$$\frac{\sin 51^\circ}{20} = \frac{\sin \beta}{21}$$

$$\frac{\sin 51^\circ}{20} = \frac{\sin 74.31^\circ}{c}$$

$$\frac{21 \sin 51^\circ}{20} = \frac{20 \sin \beta}{20}$$

$$\frac{c \sin 51^\circ}{\sin 51^\circ} = \frac{20 \sin 74.31^\circ}{\sin 51^\circ}$$

$$\sin^{-1}\left(\frac{21 \sin 51^\circ}{20}\right) = \beta = 54.69^\circ$$

$$c = 24.78$$

$$\gamma = 180^\circ - 51^\circ - 54.69^\circ = 74.31^\circ$$

B) Solve the triangle specified by $\alpha = 65^\circ 30'$, $a = 12.4$, $b = 9.6$.

$$h = b \sin \alpha = 9.6 \sin 65.5^\circ = 8.74$$

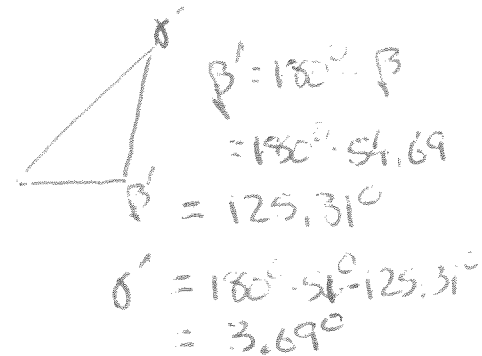
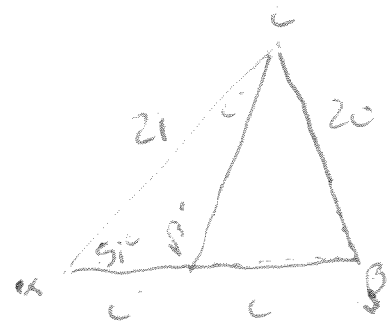
$a > b$
1 triangle

$$\frac{\sin 65.5^\circ}{12.4} = \frac{\sin \beta}{9.6}$$

$$\frac{9.6 \sin 65.5^\circ}{12.4} = \frac{12.4 \sin \beta}{12.4}$$

$$\sin^{-1}\left(\frac{9.6 \sin 65.5^\circ}{12.4}\right) = \beta = 44.79^\circ$$

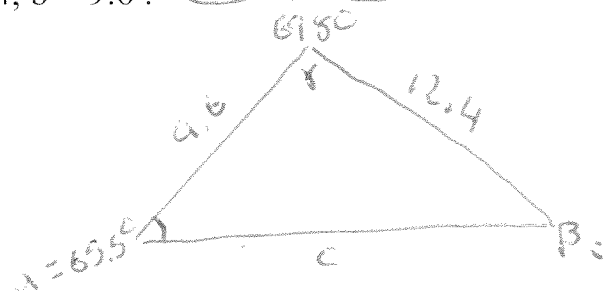
$$\gamma = 180^\circ - 65.5^\circ - 44.7^\circ = 69.8^\circ$$



$$\frac{\sin 51^\circ}{20} = \frac{\sin 3.69^\circ}{c}$$

$$\frac{c \sin 51^\circ}{\sin 51^\circ} = \frac{20 \sin 3.69^\circ}{\sin 51^\circ}$$

$$c = 1.66$$



$$\frac{\sin 65.5^\circ}{12.4} = \frac{\sin 69.8^\circ}{c}$$

$$c \sin 65.5^\circ = 12.4 \sin 69.8^\circ$$

$$c = \frac{12.4 \sin 69.8^\circ}{\sin 65.5^\circ}$$

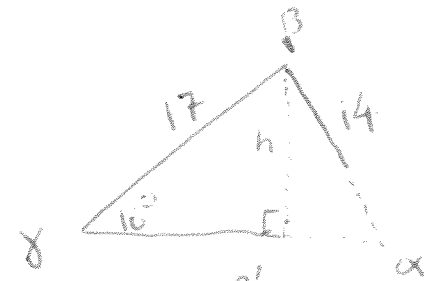
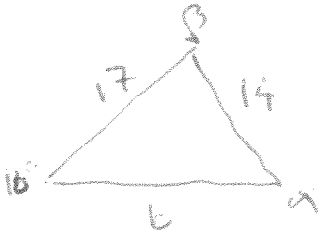
$$c = 12.79$$

C) Solve the triangle specified by $\gamma = 16^\circ$, $a = 17$, $c = 14$.

$$h = 17 \sin 16^\circ = 4.69$$

$$h < c < a$$

two triangles



$$\frac{\sin 16^\circ}{14} = \frac{\sin \alpha}{17}$$

$$\frac{\sin 16^\circ}{14} = \frac{\sin 144.45^\circ}{b}$$

$$\frac{17 \sin 16^\circ}{14} = \frac{14 \sin \alpha}{14}$$

$$\frac{b \sin 16^\circ}{\sin 16^\circ} = \frac{14 \sin 144.45^\circ}{\sin 16^\circ}$$

$$\sin^{-1}\left(\frac{17 \sin 16^\circ}{14}\right) = \alpha = 19.55^\circ$$

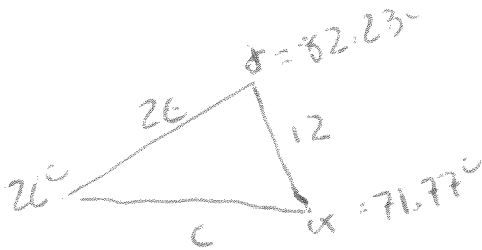
$$\beta = 180^\circ - 19.55^\circ - 16^\circ = 144.45^\circ$$

$$b = 29.53$$

D) Solve the triangle specified by $\beta = 26^\circ$, $a = 26$, $b = 12$.

$$h = 26 \sin 26^\circ = 11.40$$

2 triangles $h < b < a$



$$\frac{\sin 26^\circ}{12} = \frac{\sin \alpha}{26}$$

$$\frac{\sin 26^\circ}{12} = \frac{\sin 82.23^\circ}{c}$$

$$\frac{26 \sin 26^\circ}{12} = \frac{12 \sin \alpha}{12}$$

$$\frac{c \sin 26^\circ}{\sin 26^\circ} = \frac{12 \sin 82.23^\circ}{\sin 26^\circ}$$

$$\sin^{-1}\left(\frac{26 \sin 26^\circ}{12}\right) = \alpha = 71.77^\circ \quad c = 27.12$$

$$\delta = 180^\circ - 71.77^\circ - 26^\circ = 82.23^\circ$$

$$\frac{\sin 16^\circ}{14} = \frac{\sin 3.55^\circ}{b'}$$

$$\frac{b' \sin 16^\circ}{\sin 16^\circ} = \frac{14 \sin 3.55^\circ}{\sin 16^\circ}$$

$$b' = 3.14$$

$$\frac{\sin 26^\circ}{12} = \frac{\sin 45.77^\circ}{c'}$$

$$\frac{c' \sin 26^\circ}{\sin 26^\circ} = \frac{12 \sin 45.77^\circ}{\sin 26^\circ}$$

$$c' = 19.61$$