Name Mine Date

Math 112: #24 A/B/C/D

A) Use the half angle formulas to find the exact value of cos 7.5°. Simplify only to the point where your expression no longer contains any explicit trigonometric functions.

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$ ⊕ 7.5 is in I

$\theta = 15°$ $\frac{15°}{2} = 7.5°$

$\cos 7.5° = \sqrt{\frac{1+\cos 15°}{2}}$ $\cos\frac{30°}{2} = 15°$ $\theta' = 30°$

$= \sqrt{\frac{1+\sqrt{\frac{1+\cos 30°}{2}}}{2}}$ $\cos 30° = \frac{\sqrt{3}}{2}$

ok to stop here

$= \sqrt{\frac{1+\sqrt{\frac{1+\sqrt{3}/2}{2}}}{2}} \cdot \frac{2}{2}$

$= \sqrt{\frac{1+\sqrt{\frac{2+\sqrt{3}}{4}}}{2}}$ $= \sqrt{\frac{1+\frac{1}{2}\sqrt{2+\sqrt{3}}}{2}} \cdot \frac{2}{2} = \sqrt{\frac{2+\sqrt{2+\sqrt{3}}}{4}}$

B) Use the half angle formulas to find the exact value of tan 7.5°. Simplify only to the point where your expression no longer contains any explicit trigonometric functions. $\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$ $\theta = 15°$ $\frac{15°}{2} = 7.5°$

$= \frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{3}}}$

$\tan 7.5° = \sqrt{\frac{1-\cos 15°}{1+\cos 15°}}$ $\theta' = 30°$ $\frac{30°}{2} = 15°$

7.5°, 15° & 30° are in I so ⊕

$\tan 7.5° = \sqrt{\frac{1-\sqrt{\frac{1+\cos 30°}{2}}}{1+\sqrt{\frac{1+\cos 30°}{2}}}}$ $\cos 30° = \frac{\sqrt{3}}{2}$

$= \sqrt{\frac{1-\sqrt{\frac{1+\sqrt{3}/2}{2}} \cdot \frac{2}{2}}{1-\sqrt{\frac{1+\sqrt{3}/2}{2}} \cdot \frac{2}{2}}}$ $= \sqrt{\frac{1-\sqrt{\frac{2+\sqrt{3}}{4}}}{1+\sqrt{\frac{2+\sqrt{3}}{4}}}}$ $= \sqrt{\frac{1-\frac{1}{2}\sqrt{2+\sqrt{3}}}{1+\frac{1}{2}\sqrt{2+\sqrt{3}}} \cdot \frac{2}{2}} = \sqrt{\frac{2-\sqrt{2+\sqrt{3}}}{2+\sqrt{2+\sqrt{3}}}}$

you can stop anywhere along here

C) Use the half angle formulas to find the exact value of sin 11.25°. Simplify only to the point where your expression no longer contains any explicit trigonometric functions.

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\oplus$ because $11.25°, 22.5°$ & $45°$ are all in I

$\theta = 22.5°, \quad \frac{22.5°}{2} = 11.25°$

$\theta' = 45°, \quad \frac{45°}{2} = 22.5°$

$\cos 45° = \sqrt{2}/2$

$\sin 11.25° = \sqrt{\frac{1-\cos 22.5°}{2}}$

$= \sqrt{\frac{1-\sqrt{\frac{1+\cos 45°}{2}}}{2}}$

$= \sqrt{\frac{1-\sqrt{\frac{1+\sqrt{2}/2}{2}}}{2}} \cdot \frac{2}{2} = \sqrt{\frac{1-\sqrt{\frac{2+\sqrt{2}}{4}}}{2}} = \sqrt{\frac{1-\frac{1}{2}\sqrt{2+\sqrt{2}}}{2}} \cdot \frac{2}{2} = \sqrt{\frac{2-\sqrt{2+\sqrt{2}}}{4}}$

any of these would be fine

$= \frac{1}{2}\sqrt{2-\sqrt{2+\sqrt{2}}}$

B) Use the half angle formulas to find the exact value of ~~sin~~ cos 11.25°. Simplify only to the point where your expression no longer contains any explicit trigonometric functions.

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\theta = 22.5°, \quad \frac{\theta}{2} = 11.25$

$11.25°, 22.5°$ & $45°$ are all in I so $\oplus$

$\theta' = 45° \quad \frac{\theta'}{2} = 22.5°$

$\cos 45° = \frac{\sqrt{2}}{2}$

$\cos 11.25 = \sqrt{\frac{1+\cos 22.5°}{2}}$

$= \sqrt{\frac{1+\sqrt{\frac{1+\cos 45°}{2}}}{2}}$

$= \sqrt{\frac{1+\sqrt{\frac{1+\sqrt{2}/2}{2}}}{2}} \cdot \frac{2}{2} = \sqrt{\frac{1+\sqrt{\frac{2+\sqrt{2}}{4}}}{2}} = \sqrt{\frac{1+\frac{1}{2}\sqrt{2+\sqrt{2}}}{2}} \cdot \frac{2}{2}$

Any of these would be fine.

$= \sqrt{\frac{2+\sqrt{2+\sqrt{2}}}{4}}$

$= \frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}$