Name Mine Date ______________

Math 112: #23 A/B/C

A) Use the difference identity $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

to re-express $f(x) = 3\cos x + 4\sin x$

as $f(x) = A\cos(x - \alpha)$

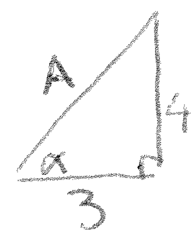
That is, find $A > 0$ and α for which $A\cos(x - \alpha) = 3\cos x + 4\sin x$

$$\frac{\cancel{A}\cos(x-\alpha)}{\cancel{A}} = \frac{3\cos x}{A} + \frac{4\sin x}{A}$$

$$\cos(\cancel{x}-\alpha) = \frac{3}{A}\cos x + \frac{4}{A}\sin x$$

$$\cos x\cos\alpha + \sin x\sin x = \frac{3}{A}\cos x + \frac{4}{A}\sin x$$

So... $\cos\alpha = \frac{3}{A}$ and $\sin\alpha = \frac{4}{A}$



$$A^2 = 3^2 + 4^2$$

$$A = \sqrt{9+16} = 5$$

$$\tan\alpha = \frac{4}{3}$$

$$\tan^{-1}(4/3)\alpha$$

$$5\cos\left(x - \tan^{-1}(4/3)\right) = 3\cos x + 4\sin x$$

B) Use the difference identity $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

to re-express $f(t) = -5\cos t + 12\sin t$

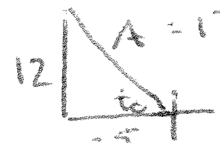
as $f(t) = A\cos(t - t_0)$

That is, find $A > 0$ and t_0 for which $A\cos(t - t_0) = -5\cos t + 12\sin t$

$$\frac{\cancel{A}\cos(t-t_0)}{\cancel{A}} = \frac{-5\cos t}{A} + \frac{12\sin t}{A}$$

$$\cos(t-t_0) = \underline{\cos t}\cos t_0 + \underline{\sin t}\sin t_0 = \frac{-5}{A}\underline{\cos t} + \frac{12}{A}\underline{\sin t}$$

$$\Rightarrow \cos t_0 = \frac{-5}{A} \qquad \sin t_0 = \frac{12}{A}$$



$$13\cos\left(t - \tan^{-1}\left(\frac{12}{-5}\right)\right) \quad \longleftarrow \quad \tan\alpha = \frac{12}{-5} \qquad A^2 = (-5)^2 + 12^2$$

$$\tan^{-1}\left(-\frac{12}{5}\right) = \alpha \qquad A = \sqrt{169} = 13$$

or

$$13\cos\left(t + \tan^{-1}\left(\frac{12}{5}\right)\right) \quad \text{since} \quad \tan\left(\frac{12}{-5}\right) = -\tan\left(\frac{12}{5}\right)$$

C) Use the difference identity $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

to re-express $f(t) = 6\cos t - 8\sin t$

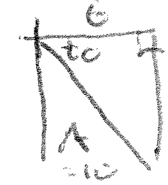
as $f(t) = A\cos(t - t_0)$

That is, find $A > 0$ and t_0 for which $A\cos(t - t_0) = 6\cos t - 8\sin t$

$$A\cos(t-t_0) = \frac{6\cos t}{A} - \frac{8\sin t}{A}$$

$$\cos(t-t_0) = \underline{\cos t}\cos t_0 + \sin t\sin t_0 = \frac{6}{A}\underline{\cos t} - \frac{8}{A}\underline{\sin t}$$

$$\text{so } \cos t_0 = \frac{6}{A} \text{ and } \sin t_0 = \frac{-8}{A} \longrightarrow$$



$$A = \sqrt{6^2 + (-8)^2} \qquad A = 10$$

$$\therefore\ 10\cos\left(t - \tan^{-1}\left(-\tfrac{4}{3}\right)\right) = 6\cos t - 8\sin t \qquad \tan t_0 = \frac{-8}{6} = -\frac{4}{3}$$

$$\text{or } = 10\cos\left(t + \tan^{-1}\left(\tfrac{4}{3}\right)\right) \qquad \tan^{-1}\left(-\tfrac{4}{3}\right) = t_0$$

$$\text{because } \tan\left(-\tfrac{4}{3}\right) = -\tan\left(\tfrac{4}{3}\right)$$