Name __Mike__   Date _____

Math 112: #22 A/B/C

A) Use the sum identity $\quad\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

and difference identity $\quad\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

to verify the identity $\quad\dfrac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}=\dfrac{\cot\alpha+\tan\beta}{\cot\alpha-\tan\beta}$

$$\frac{(\cos\alpha\cos\beta+\sin\alpha\sin\beta)/\sin\alpha\cos\beta}{(\cos\alpha\cos\beta-\sin\alpha\sin\beta)/\sin\alpha\cos\beta}=\frac{\frac{\cos\alpha\cos\beta}{\sin\alpha\cos\beta}+\frac{\sin\alpha\sin\beta}{\sin\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\sin\alpha\cos\beta}-\frac{\sin\alpha\sin\beta}{\sin\alpha\cos\beta}}$$

$$\frac{\cot\alpha+\tan\beta}{\cot\alpha-\tan\beta}\quad\Leftarrow=\frac{\frac{\cos\alpha}{\sin\alpha}+\frac{\sin\beta}{\cos\beta}}{\frac{\cos\alpha}{\sin\alpha}-\frac{\sin\beta}{\cos\beta}}$$

B) Use the difference identity $\quad\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$,

to verify the identity $\quad\tan(\frac{\pi}{4}-\gamma)=\dfrac{1-\tan\gamma}{1+\tan\gamma}$

$$\frac{\tan\frac{\pi}{4}-\tan\gamma}{1+\tan\frac{\pi}{4}\tan\gamma}=\frac{1-\tan\gamma}{1+1\cdot\tan\gamma}=\frac{1-\tan\gamma}{1+\tan\gamma}$$

C) Use the sum identity $\quad\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

and difference identity $\quad\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

to verify the identity $\quad\dfrac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}=\dfrac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}$

$$\frac{(\sin\alpha\cos\beta+\cos\alpha\sin\beta)/\cos\alpha\cos\beta}{(\sin\alpha\cos\beta-\cos\alpha\sin\beta)/\cos\alpha\cos\beta}=\frac{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}+\frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}-\frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

$$\frac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}\Leftarrow=\frac{\frac{\sin\alpha}{\cos\alpha}+\frac{\sin\beta}{\cos\beta}}{\frac{\sin\alpha}{\cos\alpha}-\frac{\sin\beta}{\cos\beta}}$$