Name Aline Date ____________

Math 112: #21 A/B/C/D/E/F

A) Use the identity $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$, along with known values of sine and cosine for special angles, to obtain the exact value, simplified to a single fraction, for

$$\sin\frac{5\pi}{12} = \sin\left(\frac{2\pi}{12}+\frac{3\pi}{12}\right) = \sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$$

$$= \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \cos\frac{\pi}{6}\sin\frac{\pi}{4}$$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$$

B) Use the identity $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$, along with known values of sine and cosine for special angles, to obtain the exact value, simplified to a single fraction, for

$$\sin 15° = \sin(45° - 30°)$$

$$= \sin 45° \cos 30° - \cos 45° \sin 30°$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

C) Use the identity $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$, along with known values of sine and cosine for special angles, to obtain the exact value, simplified to a single fraction, for

$$\cos 75° = \cos(30° + 45°)$$

$$= \cos 30° \cos 45° - \sin 30° \sin 45°$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

D) Use the identity cos(α − β) = cos α cos β + sin α sin β, along with known values of sine and cosine for special angles, to obtain the exact value, simplified to a single fraction, for

$$\cos\frac{\pi}{12} = \cos\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right) = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$$

E) Use the identity $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$, along with known values of tangent for special angles, to obtain the exact value, simplified to a single fraction, for

$$\tan\frac{5\pi}{12} = \tan\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right) = \tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$$

$$= \frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{6}\tan\frac{\pi}{4}} = \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3}\cdot 1}$$

$$= \frac{\frac{\sqrt{3}}{3} + 1}{1 - \sqrt{3}/3}\cdot\frac{3}{3} = \frac{\sqrt{3}+3}{3-\sqrt{3}}\cdot\frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{3\sqrt{3}+3+9+3\sqrt{3}}{9-3} = \frac{12+6\sqrt{3}}{6} = \boxed{2+\sqrt{3}}$$

F) Use the identity $\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$, along with known values of tangent for special angles, to obtain the exact value, simplified to a single fraction, for

$$\tan 15° = \tan(45° - 30°)$$

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45°\tan 30°} = \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}\cdot\frac{3}{3} = \frac{3-\sqrt{3}}{3+\sqrt{3}}$$

$$\frac{3-\sqrt{3}}{3+\sqrt{3}}\cdot\frac{3-\sqrt{3}}{3-\sqrt{3}} = \frac{9-3\sqrt{3}-3\sqrt{3}+3}{9-3} = \frac{12-6\sqrt{3}}{6} = 2-\sqrt{3}$$