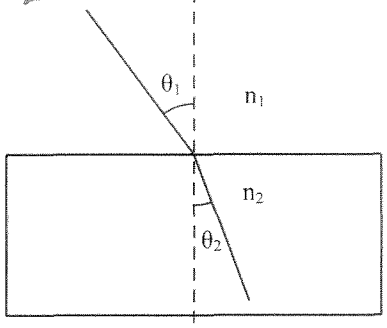
Name Minc Date _____

Math 112: #16 I/II

I) Light passing through the boundary from a substance with index of refraction n_1 to a substance with index of refraction n_2 experiences a direction change governed by Snell's law, which states that

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ where θ_1 and θ_2 are always positive



a) Water has an $n_1 = 1.33$ and glass has $n_2 = 1.52$. Estimate θ_2 to two decimal places given that a ray of light enters the glass with $\theta_1 = 40°$.

$$\frac{1.33\sin 40^\circ}{1.52} = \frac{1.52\sin\theta_2}{1.52}$$

$$0.562 = \sin\theta_2$$

$$\sin^{-1}(0.562) = \theta_2 = 34.22^\circ$$

b) Now reverse the situation and consider light exiting the glass. At what angle θ_2, will $\theta_1 = 90°$?

$$1.33\sin 90^\circ = 1.52\sin\theta_2$$

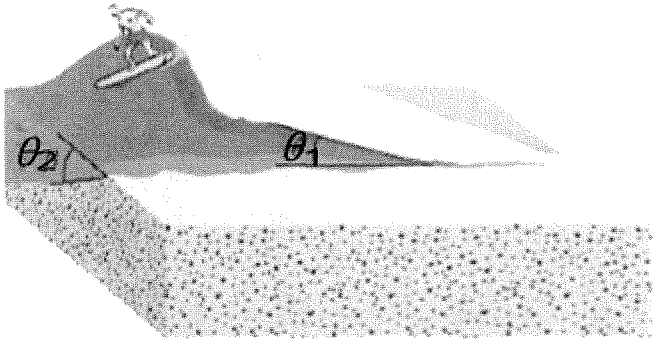
$$\frac{1.33\sin 90^\circ}{1.52} = \sin\theta_2$$

$$\sin^{-1}\left(\frac{1.33}{1.52}\right) = \theta_2 = 61.04^\circ$$

II) For a wave to be surfable, it can't break all at once. A wave has a surfable shoulder if it hits the shoreline at an angle θ given by

$$\theta_1 = \sin^{-1}\left(\frac{1}{(2n+1)\tan\theta_2}\right)$$

where θ_2 is the angle at which the beach slopes down and where n=1, 2, 3, •••



a) For $\theta_2 = 10°$, find θ_1 when n = 3

$\theta_1 = \sin^{-1}\left(\frac{1}{(2(3)+1)\tan 10}\right)$

$= \sin^{-1}\left(\frac{1}{7\tan 10}\right)$

$= \sin^{-1}\left(\frac{1}{1.234}\right)$

$= \sin^{-1}(0.810) = 54.11°$

b) For $\theta_2 = 15°$, find θ_1 when n = 2, 3, and 4. Explain why the formula does not give a value for θ_1 when n = 0 or 1.

n=2 $\theta_1 = \sin^{-1}\left(\frac{1}{5\tan 15}\right) = \sin^{-1}\left(\frac{1}{1.340}\right) = 42.24°$

n=3 $\theta_1 = \sin^{-1}\left(\frac{1}{7\tan 15}\right) = \sin^{-1}\left(\frac{1}{1.876}\right) = 32.21°$

n=4 $\theta_1 = \sin^{-1}\left(\frac{1}{9\tan 15}\right) = \sin^{-1}\left(\frac{1}{2.412}\right) = 24.50°$

n=0 $\theta_1 = \sin^{-1}\left(\frac{1}{1\tan 15}\right) = \sin^{-1}\left(\frac{1}{3.27}\right)$

n=1 $\theta_1 = \sin^{-1}\left(\frac{1}{3\tan 15}\right) = \sin^{-1}\left(\frac{1}{0.80}\right)$

$\sin^{-1}$ has Domain [-1, 1]. Both of the will be >1