Name Nhne Date _____

Math 112: #15 A/B/C/D

A) Find the domain and range of $f(x) = 3\sin^{-1}(4x-1) + 2$.

Answers need to be given in set notation: ex: $\{x \in \Re \mid -9 < x \leq 11\}$ *or* $(-9, 11]$

Domain: $\sin^{-1}x \Rightarrow [-1, +1] \;:\; -1 \leq x \leq 1$

Find $-1 \leq 4x-1 \leq 1$

$0 \leq 4x \leq 2$

$0 \leq x \leq \frac{1}{2}$ ← can't leave it in this form

$[0, \frac{1}{2}]$ or $\{x \in \mathbb{R} \mid 0 \leq x \leq \frac{1}{2}\}$

Range: $\sin^{-1}x \Rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}] \;:\; -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$3(-\frac{\pi}{2}) + 2 \leq y \leq 3(\frac{\pi}{2}) + 2$

$2 - \frac{3\pi}{2} \leq y \leq 2 + \frac{3\pi}{2}$

$[2 - \frac{3\pi}{2}, 2 + \frac{3\pi}{2}]$ or $\{y \in \mathbb{R} \mid 2 - \frac{3\pi}{2} \leq y \leq 2 + \frac{3\pi}{2}\}$

B) Find the domain and range of $f(x) = 5\cos^{-1}(2x+3) - 4$.

Domain: $\cos^{-1} \Rightarrow [-1, 1] \;:\; -1 \leq x \leq 1$

Find $-1 \leq 2x+3 \leq 1$

$-4 \leq 2x \leq -2$

$-2 \leq x \leq -1$

$[-2, -1]$ or $\{x \in \mathbb{R} \mid -2 \leq x \leq -1\}$

Range: $\cos^{-1}x \Rightarrow [0, \pi] \;:\; 0 \leq y \leq \pi$

$5(0) - 4 \leq y \leq 5(\pi) - 4$

$-4 \leq y \leq 5\pi - 4$

$[-4, 5\pi - 4]$ or $\{y \in \mathbb{R} \mid -4 \leq y \leq 5\pi - 4\}$

C) Find the domain and range of $f(x) = 2\tan^{-1}(3x+1) - 5$.

Domain $\tan^{-1}x \Rightarrow [-\infty, \infty] : -\infty \le x \le \infty$

find $-\infty \le 3x+1 \le \infty$

$\frac{-1-\infty}{3} \le x \le \frac{-1+\infty}{3}$

$\left[\frac{-1-\infty}{3}, \frac{-1+\infty}{3}\right]$

or

$\left\{x \in \mathbb{R} \mid \frac{-1-\infty}{3} \le x \le \frac{-1+\infty}{3}\right\}$

Range $\tan^{-1}x \Rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] : -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$2\left(-\frac{\pi}{2}\right)+5 \le y \le 2\left(\frac{\pi}{2}\right)+5$

$-\pi+5 \le y \le \pi+5$

$[5-\pi, 5+\pi]$

or

$\{y \in \mathbb{R} \mid 5-\pi \le y \le 5+\pi\}$

D) Find the domain and range of $f(x) = 4\cos^{-1}(3x-2) + 6$.

Domain: $\cos^{-1}x \Rightarrow [-1, 1] : -1 \le x \le 1$

$-1 \le 3x-2 \le 1$

$1 \le 3x \le 3$

$1/3 \le x \le 1$

$[1/3, 1]$ or

$\{x \in \mathbb{R} \mid 1/3 \le x \le 1\}$

Range: $\cos^{-1}x \Rightarrow [0, \pi]$ $0 \le y \le \pi$

$4(0)+6 \le y \le 4(\pi)+6$

$6 \le y \le 4\pi+6$

$[6, 4\pi+6]$

or $\{y \in \mathbb{R} \mid 6 \le y \le 4\pi+6\}$