

Math 112: #14 A/B/C/D/E/F

A) Evaluate $\sin^{-1}(\sin(\frac{5\pi}{4}))$, and explain your answer.

$$\sin \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$$

Input and output are different because \sin^{-1} is defined between $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ and not at $\frac{5\pi}{4}$.

B) Evaluate $\cos^{-1}(\cos(\frac{2\pi}{3}))$, and explain your answer.

$$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

The ~~for~~ inverse functions essentially cancel each other out and input and output are the same.

C) Evaluate $\sin^{-1}(\cos(\frac{5\pi}{6}))$, and explain your answer.

$$\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$$

The change in angles occurs because the functions are different. The change in quadrants occurs because \sin^{-1} is not defined in the 2nd quadrant, but $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

D) Evaluate $\tan^{-1}(\tan(\frac{2\pi}{3}))$, and explain your answer.

$$\tan \frac{2\pi}{3} = -\sqrt{3}$$

$$\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$$

The input and output don't agree, even though the inverse functions cancel, because the range of \tan^{-1} does not include $\frac{2\pi}{3}$.

E) Evaluate $\cos^{-1}(\cos(\frac{7\pi}{4}))$, and explain your answer.

$$\cos \frac{7\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos^{-1}(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$$

The contradiction between the input and output exists because $\frac{7\pi}{4}$ is not in the range of \cos^{-1} .
↓
 $[0, \pi]$

F) Evaluate $\sin^{-1}(\sin(\frac{4\pi}{3}))$, and explain your answer.

$$\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\sin^{-1}(-\frac{\sqrt{3}}{2}) = -\frac{\pi}{3}$$

The contradiction exists because $\frac{4\pi}{3}$ is not in the range of \sin^{-1} , which is $[-\frac{\pi}{2}, \frac{\pi}{2}]$